# Optimization of the Efficiency of Photovoltaic Cells for Laser Light: An Application to Laser Power Beaming

$Q_L$ evaluated as a function of four variables (Eq. 5):

$$Q_L(v_g, P_L, v_0, \delta v_0) =$$

$$\frac{P_L}{2\pi h\left({v_0}^2 + {\delta v_0}^2\right)}\left[\pi v_0 + \delta v_0 \ln\left(\frac{(v_g - v_0)^2 + {\delta v_0}^2}{{v_g}^2}\right) + 2v_0 \tan^{-1}\left(\frac{v_0 - v_g}{\delta v_0}\right)\right]$$

Total output power if each photon above or equal to the band gap gives the PV cell one hole electron pair with energy equal to the band gap, $hv_g$, and $A$ is the area of the cell (Eq. 6):

$$P_{out} = hv_g A Q_L$$

Input power is power of the entire laser spectrum multiplied by the cross sectional area of the laser beam which is assumed to equal the area of the cell (Eq. 7, 8):

$$P_{in} = A\int_0^\infty S_L(v)dv = AP_L$$

Definition and evaluation of the efficiency function of the cell (Eqn 9, 10):

$$\eta_{ult}(v_0, v_g, \delta v_0) = \frac{P_{out}}{P_{in}} = \frac{hv_g Q_L}{P_L} =$$

$$\frac{v_g}{2\pi\left({v_0}^2 + {\delta v_0}^2\right)}\left[\pi v_0 + \delta v_0 \ln\left(\frac{(v_g - v_0)^2 + {\delta v_0}^2}{{v_g}^2}\right) + 2v_0 \tan^{-1}\left(\frac{v_0 - v_g}{\delta v_0}\right)\right]$$

- If band gap energy is greater than energy of laser, almost no photons have enough energy to contribute to the cell
- If band gap energy is much less than energy of laser, almost all photons will contribute to the cell but each contribution will be negligible because of low band gap energy

The following is a graph of this ultimate efficiency function when $v_0 = 500\,THz$ and $\delta v_0 = 50\,THz$ *(Figure 4)*:

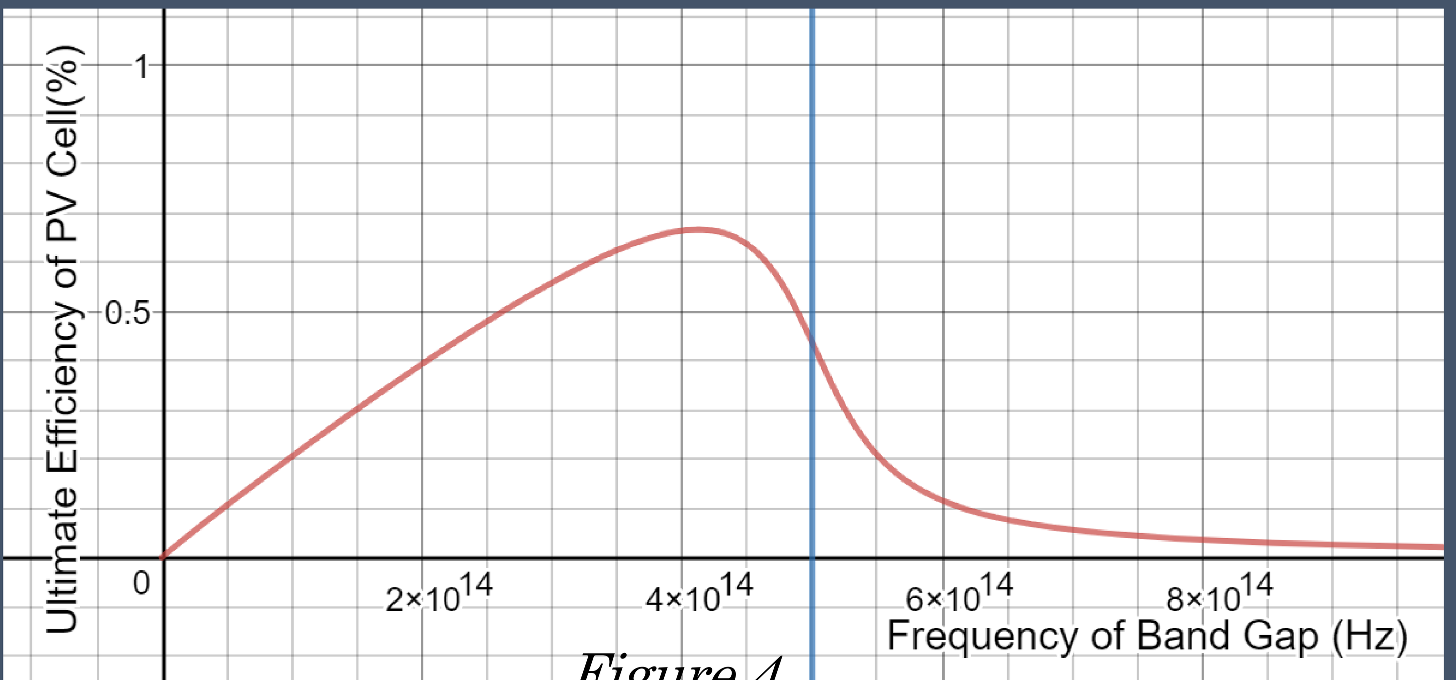


*Figure 4*

Definition of the spectral irradiance of a classic blackbody at temperature $T$ (Eq. 2).

$$S_B(v,T) = \frac{2\pi hv^3}{c^2}\frac{1}{e^{\frac{hv}{kT}} - 1}$$

Along with the output current of the cell, four processes effect the rate of appearance of holes and electrons in the cell:

| | Radiative | | Non-radiative |
|---|---|---|---|
| **Generation** | Laser source: $R_{g\gamma L}$ | Blackbody at $T_c$ source: $R_{g\gamma C}$ | $R_{g\phi}$ |
| **Recombination** | $R_{r\gamma}$ | | $R_{r\phi}$ |

According to Shockley and Queisser, by looking at a thermal equilibrium situation and a laser source situation, these rates can be defined in terms of previous quantities and the output voltage of the cell, $V$.

$$R_{g\gamma C} = Af_B Q(v_g, S_B)$$

$$R_{g\gamma L} = Af_L Q(v_g, S_L)$$

$$R_{r\gamma}(V) = R_{g\gamma C}e^{V/V_c}$$

$$R_{g\phi} = R_\phi(0)$$

$$R_{r\phi}(V) = R_\phi(0)e^{V/V_c}$$

A steady state equation when the cell is constantly producing an output current, $I$, can be defined (Eq. 19):

$$R_{g\gamma L} - R_{r\gamma} + R_{g\phi} - R_{r\phi} - \frac{I}{q} = 0$$

The steady state equation directly translates to a current equation (Eq. 22).

$$I(V) = I_{sc} + I_0(1 - e^{V/V_c})$$

where $I_0 \equiv q[R_{g\gamma C} + R_\phi(0)]$ is the reverse saturation current and $I_{sc} \equiv q[R_{g\gamma L} - R_{g\gamma C}] \cong q[R_{g\gamma L}]$ is the short circuit current.

The graph of $I(V)$ and $P = I(V) \cdot V$ is shown below (*Figure 7*).

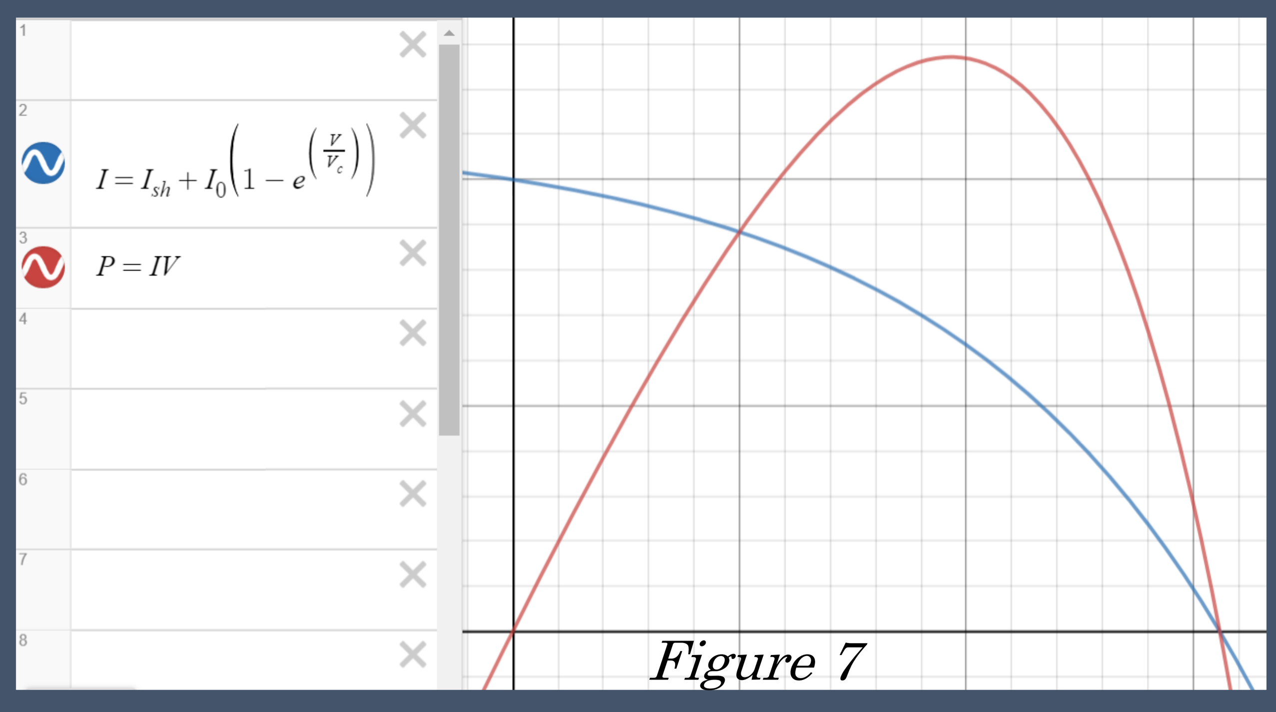


*Figure 7*

Although it is possible to directly calculate the term $I(V_{max})V_{max}$ based on the voltage resulting in maximum power, $V_{\max}$, this will give an answer in terms of some of the not well defined processes like $I_0$ and eventually $R_\phi(0)$. As Shockley and Queisser do, it is more useful to first calculate $I_{sc}V_{oc}$ in terms of $R_{g\gamma L}$ and then multiply that by the ratio of $\frac{I(V_{\max})V_{\max}}{I_{sc}V_{oc}}$, in which the $I_0$ will cancel out.

The open circuit voltage can be found by solving Eq. 22 for $I = 0$ (Eq. 28).

$$V_{oc}(T, v_g, P_L, v_0, \delta v_0, f) \cong V_c \ln\left[f_\gamma \frac{R_{g\gamma L}}{R_{g\gamma C}}\right] =$$

$$V_c \ln\left[f\frac{Q(v_g, S_L)}{Q(v_g, S_B)}\right]$$

where $f \equiv \frac{f_\gamma f_L}{f_B}$ and $f_\gamma \equiv \frac{R_{g\gamma C} - R_{r\gamma}}{R_{g\gamma C} - R_{r\gamma} + R_{g\phi} - R_{r\phi}} = \frac{R_{g\gamma C}}{R_{g\gamma C} + R_\phi(0)}$ is the fraction of all processes that is radiative.

$$\frac{d}{dV}I(V)V = I_0\left[e^{V_{oc}/V_c} - \left(1 + \frac{V}{V_c}\right)e^{V/V_c}\right]$$

By solving the equation above (Eq. 35) to find the voltage of maximum power, Shockley and Queisser found the following to be a solution (Eq. 36).

$$z_{oc} = z_m + \ln(1 + z_m)$$

where $z_{oc} \equiv \frac{V_{oc}}{V_c}$ and $z_m \equiv \frac{V_{max}}{V_c}$

The following ratio is needed to calculate the practical efficiency (Eq. 38, 39).

$$m = \frac{I(V_{\max})V_{\max}}{I_{sc}V_{oc}}$$

$$m(z_m) = \frac{{z_m}^2}{(1 + z_m - e^{-z_m})(z_m + \ln(1 + z_m))}$$

The practical efficiency can be defined as the maximum output power divided by the input power (Eq. 41, 42).

$$\eta_{prac}(T, v_g, P_L, v_0, \delta v_0, f_L, f) = \frac{I_{sc}V_{oc}}{P_{in}}\frac{I(V_{\max})V_{\max}}{I_{sc}V_{oc}}$$

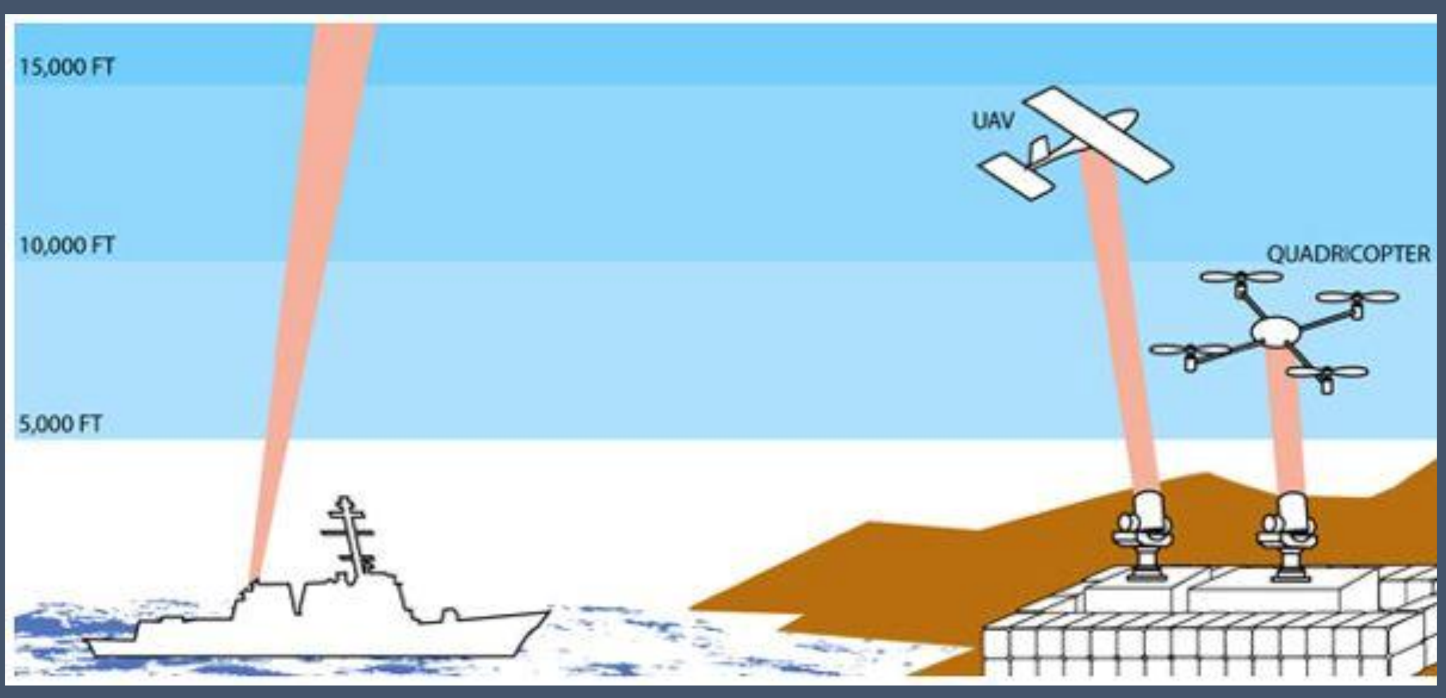
$$= mf_L\frac{V_{oc}}{V_g}\eta_{ult} =$$

$$f_L\frac{V_{oc}(T, v_g, P_L, v_0, \delta v_0, f)}{V_g}m(T, v_g, P_L, v_0, \delta v_0, f)\eta_{ult}(v_0, v_g, \delta v_0)$$

# Purpose

The purpose of this project is find the theoretical upper limit of the efficiency of laser power beaming/ power over fiber optic cables and see if it is a viable option for energy transfer.

LaserMotive's vision of laser power beaming: laser shines onto PV Cell on airborne object and transfers energy to it:




Retrieved from: https://cdn.geekwire.com/wp-content/uploads/2011/09/laserm222.jpg

# Introduction

- Laser power beaming offers a promising new way to transfer energy wirelessly
- NASA and LaserMotive have already made prototypes of drones that can stay in flight indefinitely
- Not widely used because of its relative inefficiency
- By optimizing the efficiency of conversion of monochromatic laser light into electricity with specialized PV cell, laser power beaming will become a viable option
- Will also have applications in transmitting energy through fiber optic cables

# Overview

- Shockley and Queisser found the ultimate and practical efficiency of a solar cell using a classic blackbody curve modeling the sun *(Figure 1)*
- This project replaced the blackbody spectrum with a laser spectrum and reworked all of their calculations *(Figure 2)*
- Theoretical efficiency of a PV cell with a given band gap energy and a laser with a given energy was found by first looking at the spectrum and later at the current-voltage characteristics of the cell

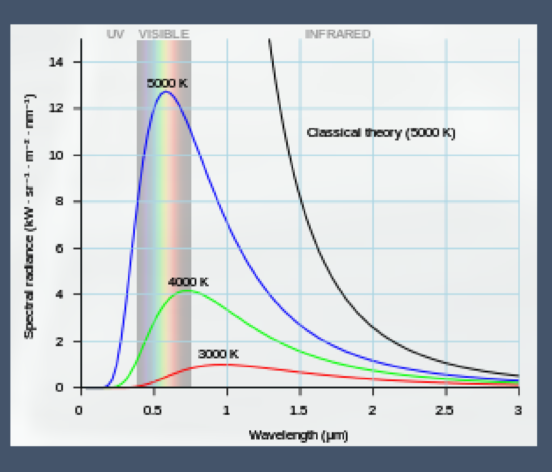



*Figure 1*:Retrieved from: https://en.wikipedia.org/wiki/Black-body_radiation#/media/File:Black_body.svg

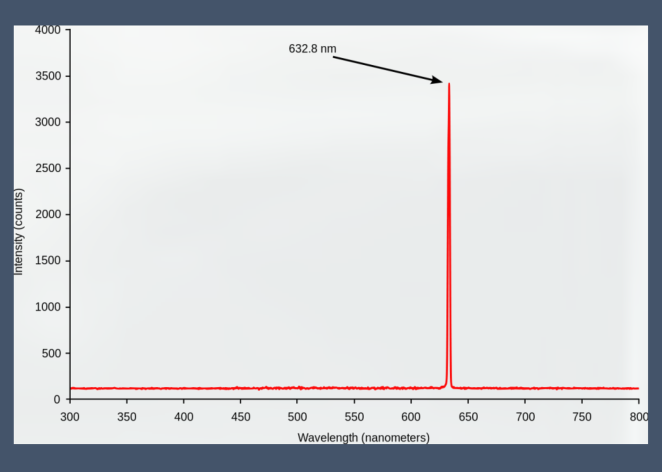



*Figure 2*: Retrieved from: https://en.wikipedia.org/wiki/Laser#/media/File:Helium_neon_laser_spectrum.svg

# Theoretical Work

Photovoltaic effect acts as a step function with the band gap of the cell and works with individual photons. The following function (Eq. 1) counts the number of photons above $\nu_{lim}$ in the spectral irradiance function $S(\nu)$ $\frac{W/m^2}{Hz}$.

$$Q(\nu_{lim}, S(\nu)) = \int_{\nu_{lim}}^{\infty} \frac{S(\nu)}{h\nu} d\nu$$

Laser light spectrum in terms of the spectral irradiance, $S_L$, the laser frequency, $\nu_0$, the power of the laser, $P_L$, and the laser linewidth, $\delta\nu_0$ (Eq. 3):

$$S_L(\nu, P_L, \nu_0, \delta\nu_0) = \frac{P_L \delta\nu_0}{\pi} \frac{1}{(\nu - \nu_0)^2 + \delta{\nu_0}^2}$$

Number of photons, $Q_L$, with frequency above the band gap frequency, $\nu_g$ (Eq. 4):

$$Q_L \equiv Q\left(\nu_g, S_L(\nu)\right) = \frac{P_L \delta\nu_0}{h\pi} \int_{\nu_g}^{\infty} \frac{1}{\nu\left[(\nu - \nu_0)^2 + \delta{\nu_0}^2\right]} d\nu$$

# Results

- Program was made to calculate all results
- Data matches ultimate efficiency graph
- Numerical approximations were used to calculate Eq. 2 and 36 (midpoint method and Riemann sum)

The ultimate and practical efficiency functions (Eq. 10, 42) were used to create the following color maps representing the efficiencies of different laser energies and band gap energies (*Figure 6, 9*).

Unless stated otherwise, all calculations were performed using $T = 300\,K$, $h\nu_g = 2.7\,eV$, $P_L = 100\,\frac{W}{m^2}$, $h\nu_0 = 2.73\,eV$, $\delta\nu_0 = 10\,GHz$, $f = f_L = 0.95$.

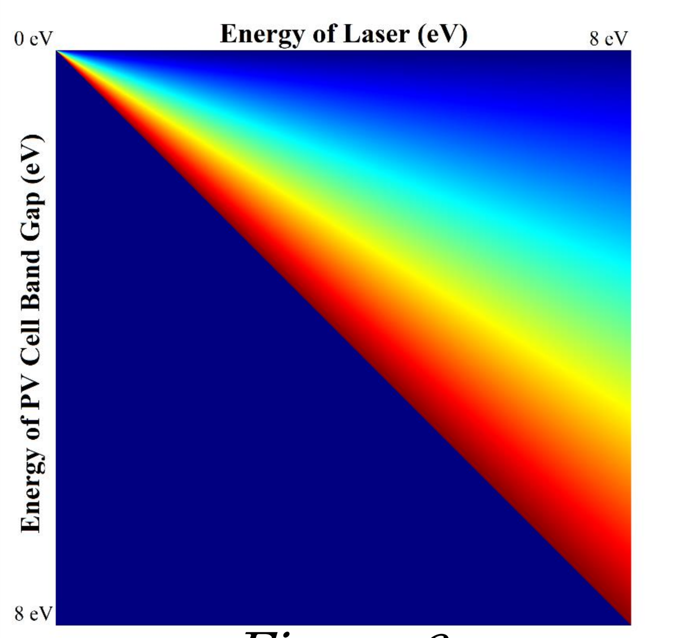



*Figure 6*

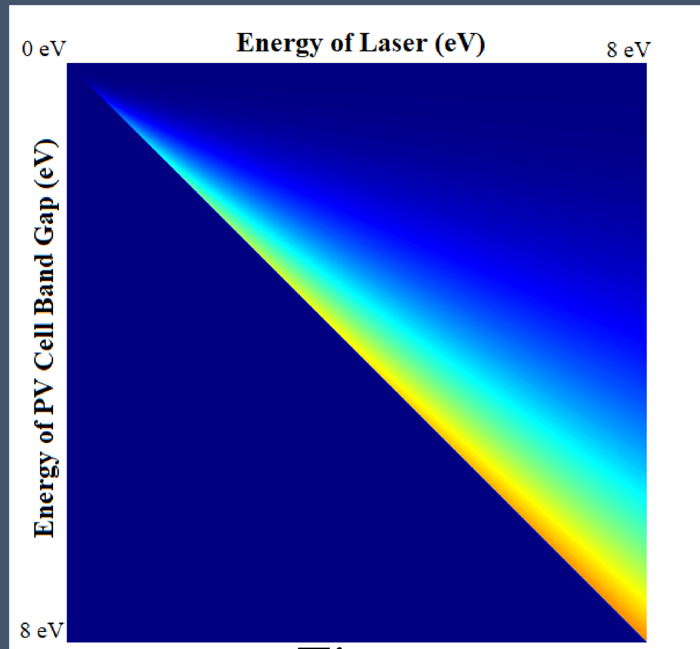
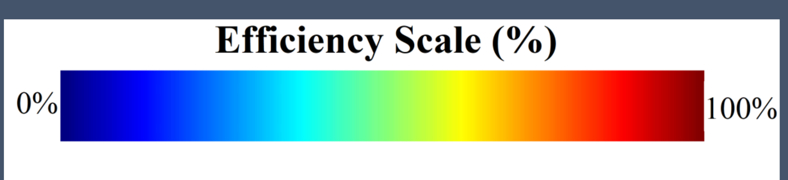



*Figure 9*




The following table shows real laser - PV Cell combinations that result in notably high efficiencies *(Table 1)*:

| Laser | PV Cell | Ultimate Efficiency | Practical Efficiency |
|---|---|---|---|
| I (0.94 eV) | $Cu_2SnS_3$ (0.91 eV) | 96.51% | 56.61% |
| InGaAsP (1.24 eV) | BAs (1.14 eV) | 91.95% | 58.68% |
| Ruby (1.79 eV) | CdSe (1.74 eV) | 97.44% | 70.27% |
| Xe+ (2.48 eV) | ZnTe (2.25 eV) | 90.74% | 74.54% |
| Xe+ (2.48 eV) | CdS (2.42 eV) | 97.59% | 80.17% |
| Ar+ (2.73 eV) | ZnSe (2.7 eV) | 98.99% | 77.79% |
| XeF-Excimer (3.51 eV) | $TiO_2$ (3.2 eV) | 91.11% | 73.41% |
| XeF-Excimer (3.51 eV) | ZnO (3.37 eV) | 95.95% | 77.89% |
| XeF-Excimer (3.51 eV) | CuCl (3.4 eV) | 96.80% | 78.68% |
| XeF-Excimer (3.51 eV) | GaN (3.44 eV) | 97.94% | 79.73% |
| N (3.68 eV) | ZnS-cubic (3.54 eV) | 96.25% | 78.64% |
| N (3.68 eV) | NiO (3.6 eV) | 97.88% | 80.15% |
| XeCl-Excimer (4.03 eV) | ZnS-hex (3.91 eV) | 97.13% | 80.34% |
| ArF-Excimer (6.42 eV) | AlN (6.28 eV) | 97.76% | 84.65% |

*Table 1*

The following graphs of the practical efficiency were also computed (*Figure 10, 11, 12*). Non-linear effects that invalidate this project's model will always limit the efficiency between 0% and 100% .

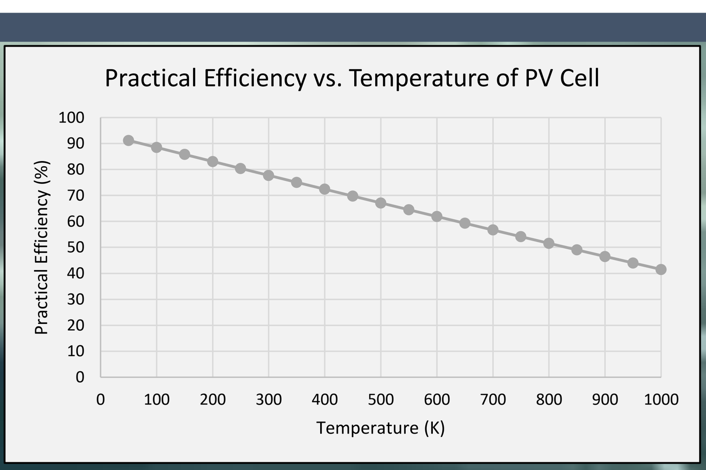



*Figure 10*

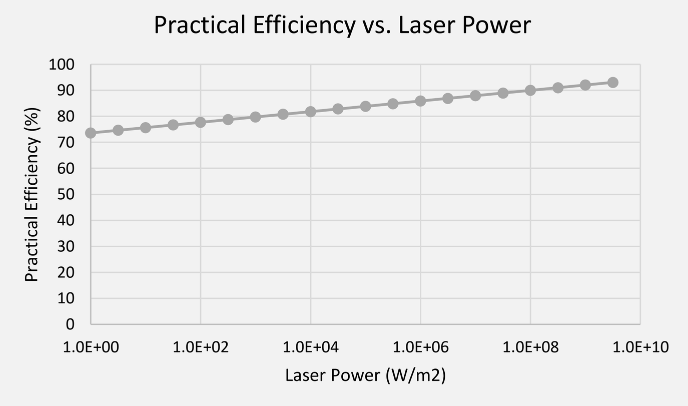



*Figure 11*

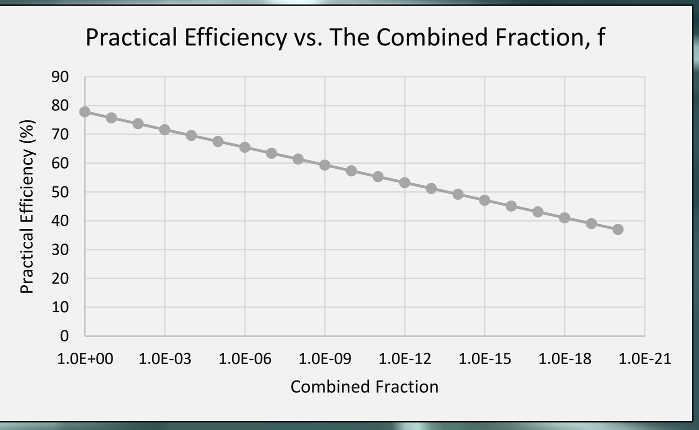



*Figure 12*

# Conclusion and Future Research

- Laser power beaming and power over fiber is a viable option for energy transfer when using certain combinations of lasers and PV cells
- Practical efficiencies of over 80% can be reached, as is the case with the argon fluoride excimer laser and the PV cell made from the semiconductor aluminum nitride
- Future researchers should theoretically model efficiency degrading factors
  - The lack of proper laser light-cell incidence
  - Electrical losses within the cell and within the circuit
  - Non-linear quantum effects.
- Future researchers can also make simulations of the model and do real life tests to show actual efficiencies reached

All images are created by the researcher unless cited otherwise