Optimization of the Efficiency of Photovoltaic Cells for Laser Light: An Application to Laser Power Beaming

Arkansas School for Mathematics, Sciences, and the Arts

Akarsh Kumar

18 February 2018

Abstract

In a vastly increasing energy dependent society, there is need for the transportation of energy quickly and efficiently. Laser power beaming and transferring energy with fiber optic cables are two methods that accomplish this. The low efficiency of converting laser light energy to electrical energy via photovoltaic cells is a problem. This project aims to optimize the efficiency of a PV cell for laser light using a theoretical approach. Shockley and Queisser's methods of a optimizing a photovoltaic cell for sunlight were used, and the blackbody spectrum of the sun was replaced with a laser spectrum. The mathematical calculations were done for this laser-PV cell system given the condition that a photon with an energy above the band gap of the PV cell may create an exciton with energy equivalent to the band gap. By considering the current-voltage characteristics of the cell, the practical efficiency equation was found as a function of the laser frequency, the band gap frequency, and 5 other independent variables. It was used to calculate the efficiencies of over 600 laser-PV cell combinations and to create a color map representing the three dimensional efficiency function. Most efficiencies were low, however, when the laser frequency was only slightly over the band gap frequency, practical efficiencies of over 80% were reached. These findings can be the start of experimentation using the laser-cell combinations highlighted in the paper and can result in a new era of energy transfer.

Introduction

Over one hundred years ago, Nikola Tesla was making incredible discoveries in the field of electromagnetics. One of his biggest aspirations was to discover a way to wirelessly transmit large amounts of energy over long distances. Transferring energy long distances wirelessly could provide humanity with vast opportunities for the future. Although Tesla was unsuccessful in finding a way to transmit energy wirelessly, the idea of wireless power transfer via lasers is still being researched and advanced to this day as a promising step toward the new generation of energy transmission.

There are two ways to use light to transfer energy: laser power beaming and power over fiber. Laser power beaming is the act of transmitting energy via laser light through the air. In most applications of laser power beaming, there is a ground-based laser that is shined onto a photovoltaic cell that converts the light energy back into electrical energy. NASA has already engineered a remote control plane that is refueled by laser power beaming [1]. The plane demonstrated it was capable of indefinite, nonstop flight by being powered by a ground-based laser constantly pointing at the photovoltaic panels on the plane [1]. NASA also hosted competitions to find different ways to power a space elevator, in which the most common way used is laser power beaming [2]. Private companies have also taken a role in this emerging field as well. For example, LaserMotive is a company dedicated to laser power beaming technology [3]. This new technology shows potential for future use in refueling airplanes while in flight and powering space elevators and devices from a distance. Laser power beaming offers exciting possibilities once thought to exist only in science fiction.

Power over fiber is the act of transferring energy through fiber optic cables. Fiber optic cables are cables designed for light to travel through, while bouncing and refracting inside the cable. Fiber optic cables have already demonstrated their effectiveness in transferring data. They are the core of the internet. Submarine communications cables are massive cables running across the floor of the ocean to connect different parts of the internet, and are in most cases, fiber optic cables. The repeater stations are electrical, so electricity must be sent via wires to power them. Power over fiber could eliminate this need. If power were transferred through fiber optic cables instead of just data, it would provide numerous advantages. It would make these wires much more safe because it does not use voltages. It would also be less prone to causing fires and other hazards electricity has. However, to transfer energy over fiber optic cables, instead of just data, there would need to be a receiver on the other side to convert the light energy back into electricity: a photovoltaic cell.

Light is effective for energy transmission because it requires no medium to travel through and does not lose energy when traveling through a vacuum. This makes light an ideal candidate for transmitting energy long distances. A laser is a device that can generate a large amount of monochromatic light that is highly directional [4]. Focused laser light can transport a great amount of energy onto a small target area. Since lasers shine in a beam with a very low divergence angle, it does not disperse or lose irradiance over long distances. These advantages make lasers the ideal tool to transfer energy.

Both laser power beaming and power over fiber depend on the efficiency of the photovoltaic cell receiving the light energy and converting it back into electricity. It is the same concept as simply collecting solar power with a solar panel, but has much more potential for growth.

Photovoltaic cells can be optimized for monochromatic light [1]. Since photovoltaic cells work by utilizing the photovoltaic effect at the most basic level, the frequency of light that causes the most electron disturbance can vary for different materials depending on the work function of the semiconductor used in the cell. If the photon does not have enough energy, it will not interact with the electrons in the semiconductor. On the other hand, if it has too much energy, it will promote the electron but the extra energy in the photon will be converted to heat in the cell. There is an optimal energy for the photon so it promotes an electron but does not create excess heat. Normal solar cells are adapted for many frequencies of light because the sun generates light of many frequencies, as shown in *Figure 1*.

Figure 1

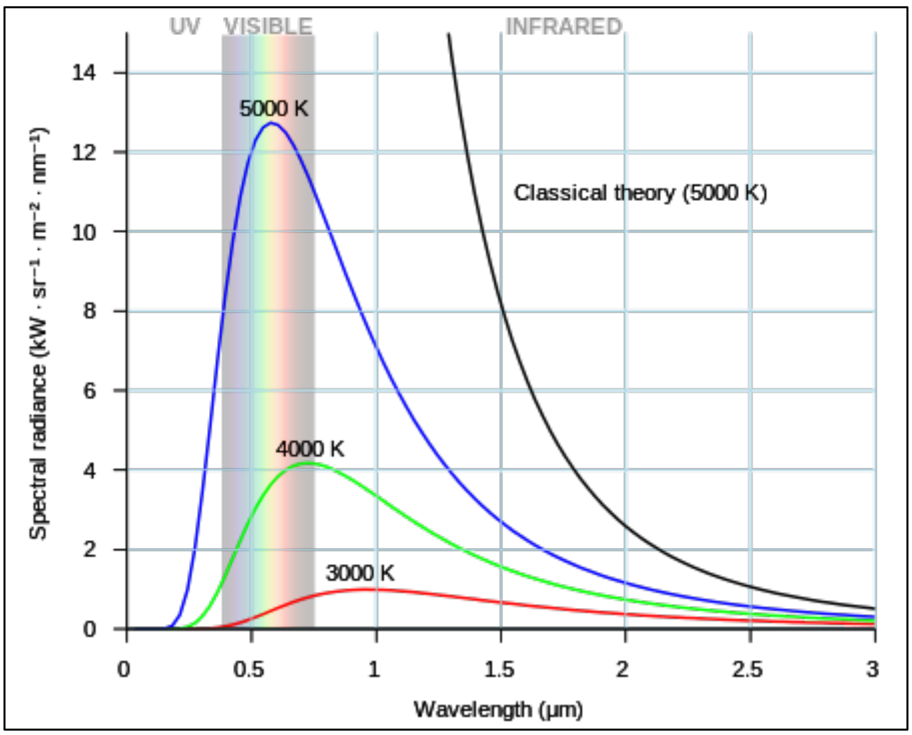


Figure 1 shows Planck's Law of Black Body Radiation illustrating the different frequencies of radiation emitted by a blackbody. Retrieved from https://upload.wikimedia.org/wikipedia/commons/thumb/1/19/Black_body.svg/300px-Black_body.svg.png

Lasers and other sources of monochromatic light have a drastically different spectrum. Their graph of intensity of light vs. frequency is extremely narrow, transmitting all of its energy in one select frequency, as seen in *Figure 2*.

Figure 2

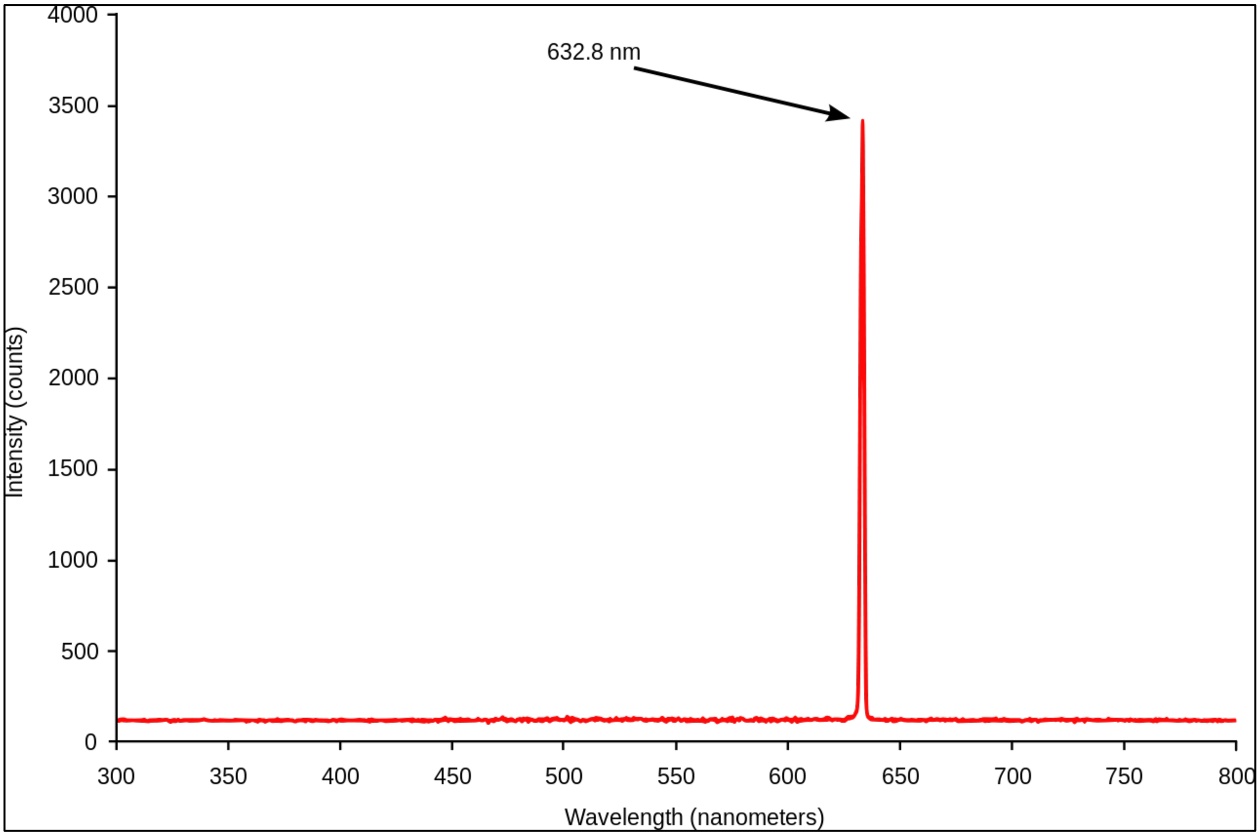


Figure 2 shows the spectrum of a helium neon laser. Retrieved from https://en.wikipedia.org/wiki/Laser#/media/File:Helium_neon_laser_spectrum.svg

If the optimal frequency causing the most electron disturbance in the cell matches the frequency of the laser light shined on it, the cell will be "tuned" to the frequency of the laser light [2]. Tuned photovoltaic cells can reach high efficiencies [2]. These tuned photovoltaic cells' efficiencies will be drastically higher than the low efficiency typically seen in a solar cell.

Many sources have reported that the most effective photovoltaic cells are made from gallium arsenide (GaAs) [1, 2, 8]. One company reported their photovoltaic cell reaching efficiencies of up to 50%, compared to the 20% seen by the average silicon solar cell [1, 5]. That same company has also made cells whose efficiencies go up with the irradiance of the light shined on them, proving to be effective with the high irradiance nature of lasers [5].

The reason gallium arsenide is considered to be a good material for photovoltaic cells is because of the way it promotes electrons. The electrons bound to the atoms in the valence band can become free electrons that can conduct electricity throughout the material in the conduction band when given a certain energy boost. GaAs has a direct bandgap between the valence band and conduction band. Direct bandgap elements allow a much easier transition for the electron to the jump through when compared to indirect bandgap elements that require the electron to move to an intermediate state before jumping. While direct band gap elements require just a photon to promote electrons, indirect bandgaps require a photon and a phonon to promote the electron. The photovoltaic effect is more efficient for direct band gap elements because it allows for easier freeing of the electrons from the atoms.

Figure 3

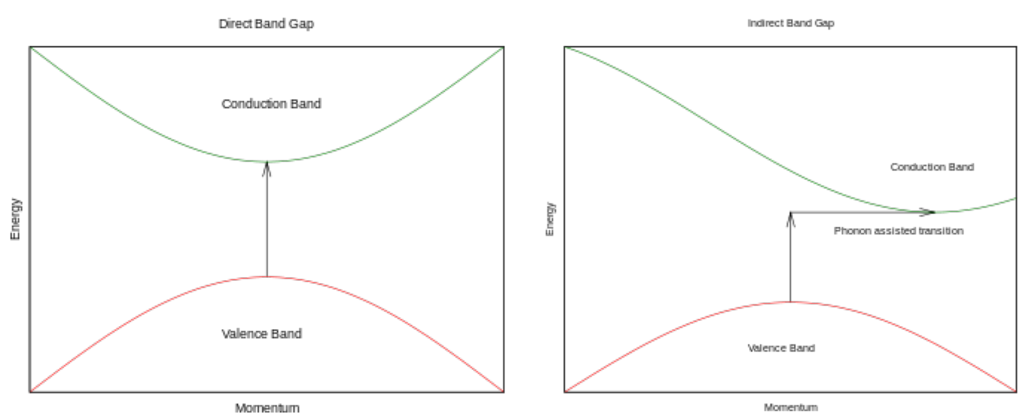


Figure 3 shows an illustration of a direct and indirect band gap. Retrieved from https://en.wikipedia.org/wiki/Direct_and_indirect_band_gaps#/media/File:Indirect_Bandgap.svg

Since the major problem holding laser power beaming and power over fiber back from becoming widespread is the low efficiencies of the photovoltaic receivers, this project will focus on optimizing photovoltaic cells for monochromatic light. Although these systems are in the early stages of development, they are an emerging field that has the potential to turn into an industry that can revolutionize how humans transfer power.

Theoretical Work: Ultimate Efficiency

The maximum efficiency of a laser-PV cell system can be calculated theoretically using the techniques of Shockley and Queisser [9]. The Shockley-Queisser limit is an efficiency limit for PV cells under sunlight that found by Shockley and Queisser using their detailed balance approach, which involves taking into account the spectral irradiance of a blackbody and the current and voltage characteristics of the cell [9]. By replacing the solar light spectrum with a laser light spectrum, the detailed balance limit of a laser-PV cell system can be found. This section will discuss the highest possible efficiency available to a PV cell given a certain laser spectrum if all available energy is utilized, taking into account only the laser spectrum and the band gap of the cell. The recombination processes of hole-electrons is not considered, only the radiative generation.

Since the band gap is a step function in that the photons with energy above the band gap contribute to the cell by creating hole-electron pairs while photons with energy below the band gap do not contribute, the following definition of a photon counting function from Shockley-Queisser is used [9].

$$Q(\nu_{lim}, S(\nu)) = \int_{\nu_{lim}}^{\infty} \frac{S(\nu)}{h\nu} d\nu \tag{1}$$

$Q(\nu_{lim}, S(\nu))$ gives the number of photons per second per unit area with frequency equal to or above ν_{lim} in the spectral irradiance function $S(\nu)$ which has units $\frac{W/m^2}{Hz}$. By dividing the spectral irradiance function by the energy of a photon, $h\nu$, it becomes a 'photon density' function rather than an 'intensity density' function. Integrating over this photon density curve will result in a number of photons.

The classic definition of a blackbody's spectral irradiance is defined by Planck's law, which gives the spectral radiance in $\frac{W/(sr\cdot m^2)}{Hz}$, multiplied by π:

$$S_B(\nu,T) = \frac{2\pi h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1} \tag{2}$$

Milonni and Eberly defined the laser light spectrum to be in the form of:

$$S_L(\nu,P_L,\nu_0,\delta\nu_0) = \frac{P_L\delta\nu_0}{\pi}\frac{1}{(\nu-\nu_0)^2+\delta{\nu_0}^2} \tag{3}$$

where $S_L(\nu,P_L,\nu_0,\delta\nu_0)$ is the spectral irradiance as a function of frequency ν, the power of the laser, P_L, the central frequency of the laser, ν_0, and the natural linewidth of the laser, $\delta\nu_0$ [10]. When divided by P_L, this curve is normalized with respect to ν.

By applying the photon counting function from Eq. (1) on the laser spectrum, it is possible to count the number of photons, Q_L, with energy above the bandgap.

$$Q_L \equiv Q\big(\nu_g,S_L(\nu)\big) = \int_{\nu_g}^{\infty}\frac{S_L(\nu)}{h\nu}d\nu = \frac{P_L\delta\nu_0}{h\pi}\int_{\nu_g}^{\infty}\frac{1}{\nu\left[(\nu-\nu_0)^2+\delta{\nu_0}^2\right]}d\nu \tag{4}$$

Evaluating this integral results in the following function of four variables:

$$Q_L\big(\nu_g,P_L,\nu_0,\delta\nu_0\big) = \frac{P_L}{2\pi h\left({\nu_0}^2+\delta{\nu_0}^2\right)}\Bigg[\pi\nu_0 + \delta\nu_0\ln\left(\frac{(\nu_g-\nu_0)^2+\delta{\nu_0}^2}{{\nu_g}^2}\right) + 2\nu_0\tan^{-1}\left(\frac{\nu_0-\nu_g}{\delta\nu_0}\right)\Bigg] \tag{5}$$

Q_L represents the number of usable photons that will generate energy in the cell given a bandgap energy. Since the ultimate efficiency is being found, it will be assumed that every single photon in Q_L will produce energy in the cell equal to the band gap energy, $h\nu_g$. In this case, the output power of the cell per unit area is the number of photons multiplied by the band gap energy, because that is how much work each photon will do in the cell. For the sake of simplification, it is assumed that the entire laser beam exactly covers the entirety of the PV cell. This simplification allows for the calculation of the overall output power of the cell to be the output power per unit area multiplied by the area of the cell, A.

$$P_{out} = h\nu_g A Q_L \tag{6}$$

Rather than simply integrating over the laser spectrum, it is required to convert the spectrum to discrete particles of light energy in calculations because each photon above the bandgap energy only contributes the bandgap energy, $h\nu_g$, to the cell, not its excess energy. For this reason, photovoltaics is highly dependent single photon-hole-electron-pair interactions.

The input power per unit area of the cell, however, is the entire set of energy coming onto the cell, which can be found by integrating over the complete laser spectrum.

$$\int_0^\infty S_L(\nu)d\nu = P_L \tag{7}$$

The total input power is this multiplied by the area of the cell, assuming the beam is not diverging considerably is maintaining a constant cross sectional area, A.

$$P_{in} = AP_L \tag{8}$$

The ultimate efficiency η_{ult} is defined to be the total output power of the cell divided by the total incident input power.

$$\eta_{ult} = \frac{P_{out}}{P_{in}} = \frac{h\nu_g Q_L}{P_L} \tag{9}$$

By plugging in the expression from Eq. (5), η_{ult} results in the following function of laser frequency, band gap frequency, and the laser linewidth.

$$\eta_{ult}(\nu_0, \nu_g, \delta\nu_0) = \frac{\nu_g}{2\pi(\nu_0{}^2 + \delta\nu_0{}^2)}\left[\pi\nu_0 + \delta\nu_0 \ln\left(\frac{(\nu_g - \nu_0)^2 + \delta\nu_0{}^2}{\nu_g{}^2}\right) + 2\nu_0 \tan^{-1}\left(\frac{\nu_0 - \nu_g}{\delta\nu_0}\right)\right] \tag{10}$$

The ultimate efficiency function will look like the following graph of efficiency versus band gap frequency below, given the values of $\nu_0 = 500\,THz$ and $\delta\nu_0 = 50\,THz$.

Figure 4: The Ultimate Efficiency Graph

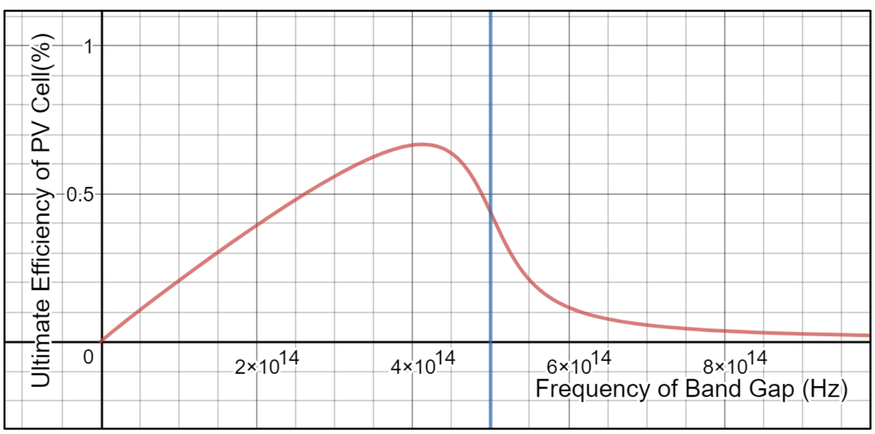


Figure 4 shows the graph of the ultimate efficiency of an ideal PV cell as a function of band gap frequency. The vertical blue line is the frequency of the laser.

The graph has a maximum, because if the band gap frequency is greater than the frequency of the laser, then almost no photons will have enough energy to contribute to the cell and if the band gap frequency is too low for the frequency of the laser, then almost all of the photons will contribute to the cell but each contribution will be close to negligible because of the low band gap energy. The excess energy of each photon will go into not useful forms of energy such as heat. The optimal band gap frequency is right below the frequency of the laser, as shown above, because it provides the perfect mix between having the majority of photons contribute and each photon contributing a reasonably large percentage of its energy.

Real Life Calculations

In practicality, the laser linewidth is almost negligible compared to the laser frequency for almost all lasers. By considering the limit of the laser spectrum as the linewidth approaches zero, the laser spectrum becomes a Dirac Delta Function multiplied by the power of the laser.

$$\lim_{\delta\nu_0 \to 0} S_L(\nu, P_L, \nu_0, \delta\nu_0) = P_L \delta(\nu - \nu_0) \tag{11}$$

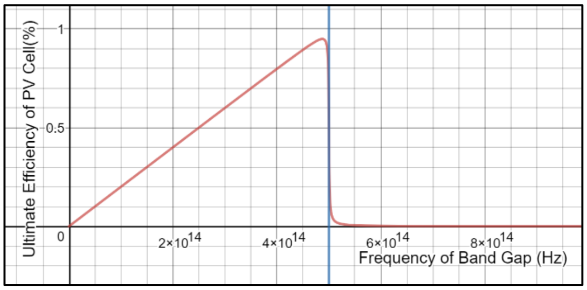
Taking the limit of the ultimate efficiency function as the linewidth approaches zero, the ultimate efficiency becomes a simple piecewise function.

$$\lim_{\delta\nu_0 \to 0} \eta_{ult}(\nu_0, \nu_g, \delta\nu_0) = \begin{cases} \nu_g/\nu_0, & \nu_g < \nu_0 \\ 0, & \nu_g \geq \nu_0 \end{cases} \tag{12}$$

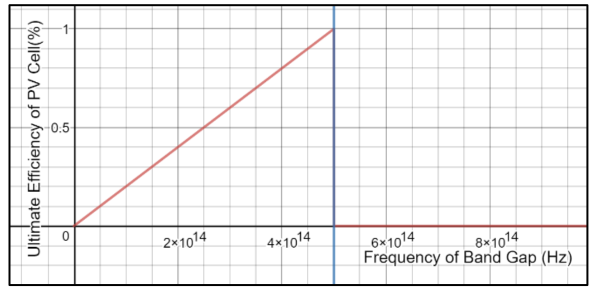
This is to be expected, as each of the photons from the laser are going to be contributing a total percentage of their energy equal to ν_g/ν_0.

The following two graphs of ultimate efficiency were found using a laser frequency of 500 THz and a linewidth of 1 THz and 1 GHz, respectively. The graph quickly approaches a straight line.

Figure 5



Linewidth of 1 THz



Linewidth of 1 GHz

Figure 5 shows the difference in ultimate efficiency graphs of low and high linewidths of the laser.

The only difference in efficiency was found when the linewidth exceeded the order of magnitude of 1 THz, which is unreasonably high for lasers. Some lasers have a linewidth of less than 10 KHz. For this reason, all data in this paper was generated using a still above-average linewidth of 10 GHz.

Since the ultimate efficiency function is a 3 dimensional function of the energy/frequency of the laser and the energy/frequency of the band gap when using a constant for the linewidth, a color map representing the 3D function was produced (*Figure 6*). The horizontal axis and vertical axis are the energy of the laser and band gap respectively in eV. The color scale maps the color output to an ultimate efficiency produced by the laser-PV cell combination as described by the ultimate efficiency function in Eq. (10).

Figure 6: Color map representation of the ultimate efficiency function (Eq. 10)

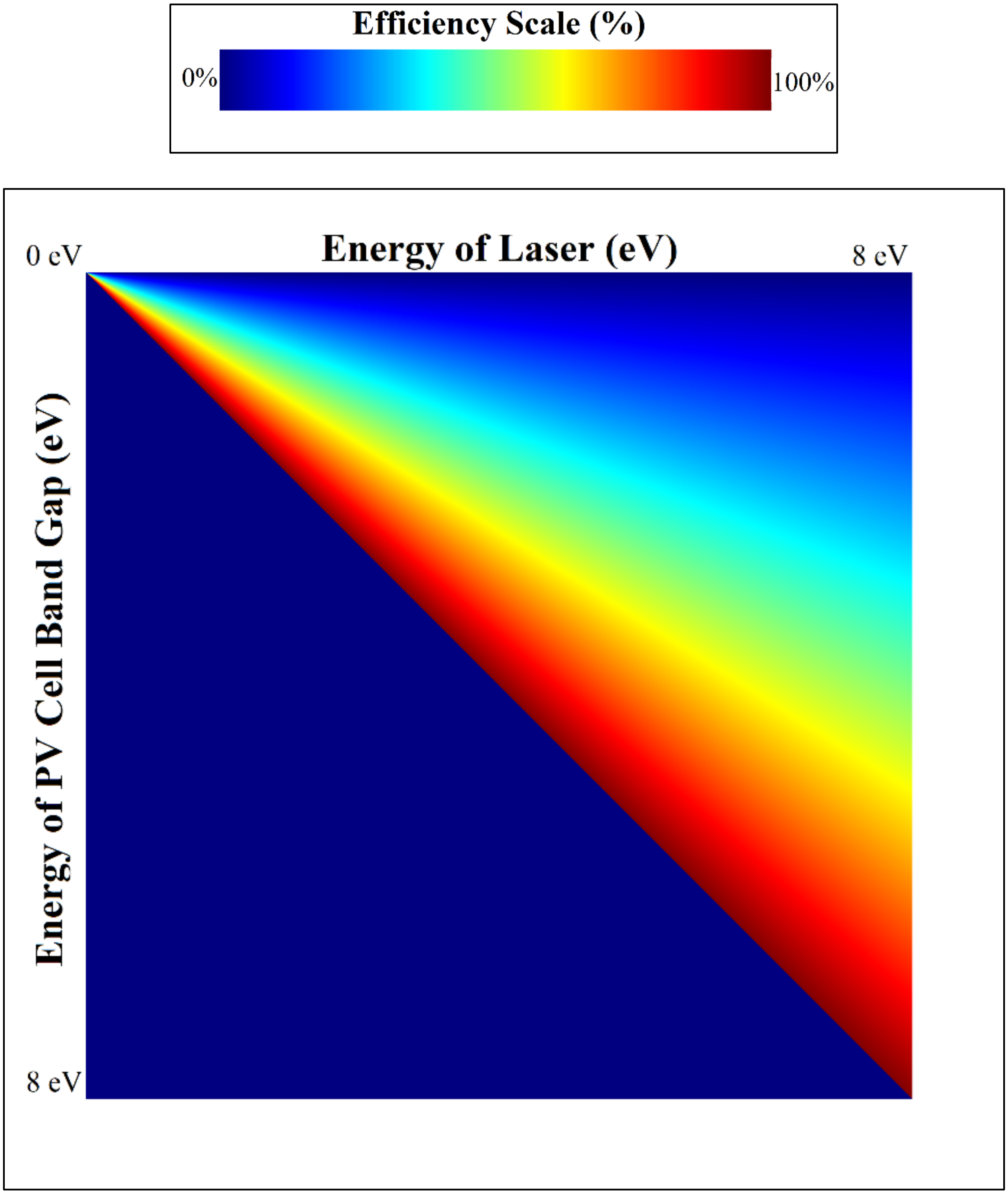


Figure 6 shows a color map representation of the 3 dimensional ultimate efficiency function (Eq. 10) using the scale shown above.

The data in *Figure 6* directly matches the graph of efficiency in *Figure 4* and *Figure 5*. Doing a cross section across a constant laser energy, the efficiency goes from a gradual increase until hitting a maximum efficiency and then suddenly dropping to zero.

Using the frequency of a laser with a semiconductor's band gap a matrix of efficiencies from real lasers versus real semiconductors can be created and evaluated to find which semiconductor-laser combination results in the optimal efficiency for the transfer of energy. This matrix was created using a program that evaluated the efficiency of every laser-semiconductor combination using a laser linewidth of 10 GHz.

Theoretical Work: Practical Efficiency

Shockley and Queisser have done significant work in calculating the practical efficiency of a photovoltaic cell under sunlight by viewing the current and voltage outputted by the cell [9]. This paper will take their calculations and rework them according to a laser spectrum being shined onto the cell. The end result is the exact equation derived by Shockley and Queisser, but with a different equation for ultimate efficiency in the previous section.

To calculate the practical efficiency of a photovoltaic cell in a steady state of a constant output current, two situations need to be considered. One is a steady state where the photovoltaic cell at a temperature T_c is at thermal equilibrium with an object fully surrounding it. That object radiates a blackbody at T_c also. The cell produces no current because the amount of energy it receives is canceled out by how much energy is radiated. The other situation is a steady state where a laser is being shined onto the photovoltaic cell which produces a constant output voltage and current.

Since the cell is being completely powered by the number of hole-electron pairs within it, the rate of generation and recombination of hole-electron pairs needs to be calculated. In both of these situations, there are five factors that affect the rate of hole-electron pairs appearing and disappearing: radiative generation of hole-electron pairs from the external blackbody/laser, radiative recombination of hole-electron pairs (resulting in photons leaving the cell), non-radiative generation of hole-electron pairs, non-radiative recombination of hole-electron pairs, and the output current of the cell constantly depleting hole-electron pairs [9]. A non-radiative process is any process that creates or abolishes pairs without releasing or absorbing a photon. This is most often through thermal activity within the cell and the interaction with phonons. To keep track of the initial four factors affecting the rate of pairs created and destroyed, the following table shows the variables used.

	Radiative		**Non-radiative**
Generation	Laser source: $R_{g\gamma L}$	Blackbody at T_c source: $R_{g\gamma C}$	$R_{g\phi}$
Recombination	$R_{r\gamma}$		$R_{r\phi}$

The radiative generation processes have the following well defined expressions, using definitions from the ultimate efficiency section of this paper.

$$R_{g\gamma C} = Af_B Q(\nu_g, S_B) \tag{13}$$

$$R_{g\gamma L} = Af_L Q(\nu_g, S_L) \tag{14}$$

In these equations, f_B and f_L represents a more realistic percentage of the photons in the blackbody spectrum and laser spectrum, respectively, with energy above the band gap that contribute to the cell. These have values known as the external quantum efficiency of a PV cell with respect to the spectrum that is shining on it which represent the percentage of incident photons that successfully create an excitation.

Shockley and Queisser have defined the other processes in terms of the thermal voltage of the cell [9]. The thermal voltage of the cell is defined to be the following expression.

$$V_c \equiv \frac{kT_c}{q} \tag{15}$$

When cell is at steady state at thermal equilibrium, the output voltage is zero. This means that the radiative recombination term, $R_{r\gamma}$ is equal to the radiative generation term, $R_{g\gamma C}$. According to Shockley and Queisser, $R_{r\gamma}$, in a situation where there is an output voltage, can be expressed as an exponential function of the output voltage based on the behavior of the densities of the holes and electrons within the material [9].

$$R_{r\gamma}(V) = R_{g\gamma C}e^{V/V_c} \tag{16}$$

When the output voltage is zero, this function will collapse to $R_{r\gamma} = R_{g\gamma C}$. A similar idea can be established with the non-radiative processes. When the voltage of the cell is zero, the non-radiative generation must be equal to the non-radiative recombination, since there are no excess hole-electron pairs produced. Since the non-radiative generation will be constant relative to voltage because it is only dependent on the thermal activity of the cell, it can be expressed as a voltage function at zero voltage.

$$R_{g\phi} = R_{\phi}(0) \tag{17}$$

Shockley and Queisser also asserted that the non-radiative recombination is an exponential function of the output voltage [9].

$$R_{r\phi}(V) = R_{\phi}(0)e^{V/V_c} \tag{18}$$

Notice how the recombination processes of the cell are constant values of the generation processes multiplied by functions that increase as the voltage increases. The recombination and generation processes are perfectly equal when voltage is equal to zero, otherwise they become unequal.

Using the five factors that contribute to the number of hole-electron pairs discussed earlier, a steady state equation for the situation of a laser can be written.

$$R_{g\gamma L} - R_{r\gamma} + R_{g\phi} - R_{r\phi} - \frac{I}{q} = 0 \tag{19}$$

It is possible to define a fraction of all of the generation and recombination within the cell that is radiative in the first steady state.

$$f_\gamma \equiv \frac{R_{g\gamma C} - R_{r\gamma}}{R_{g\gamma C} - R_{r\gamma} + R_{g\phi} - R_{r\phi}} = \frac{R_{g\gamma C}}{R_{g\gamma C} + R_\phi(0)} \tag{20}$$

This fraction may seem arbitrary, but it will help in future calculations. It is hard to determine the precise value of $R_\phi(0)$ since there exists no clear definition, so instead it is possible to just try different fractional values of f_γ.

After the solving for I in Eq. (19), the following equation is derived.

$$I = q\left[R_{g\gamma L} + \left(R_{g\gamma C} - R_{g\gamma C}\right) + R_{r\gamma} + R_{g\phi} - R_{r\phi}\right] \tag{21}$$

Using the definitions from Eq. (13), (14), (16), (17), (18) this equation can be rewritten as

$$I(V) = I_{sc} + I_0(1 - e^{V/V_c}) \tag{22}$$

where

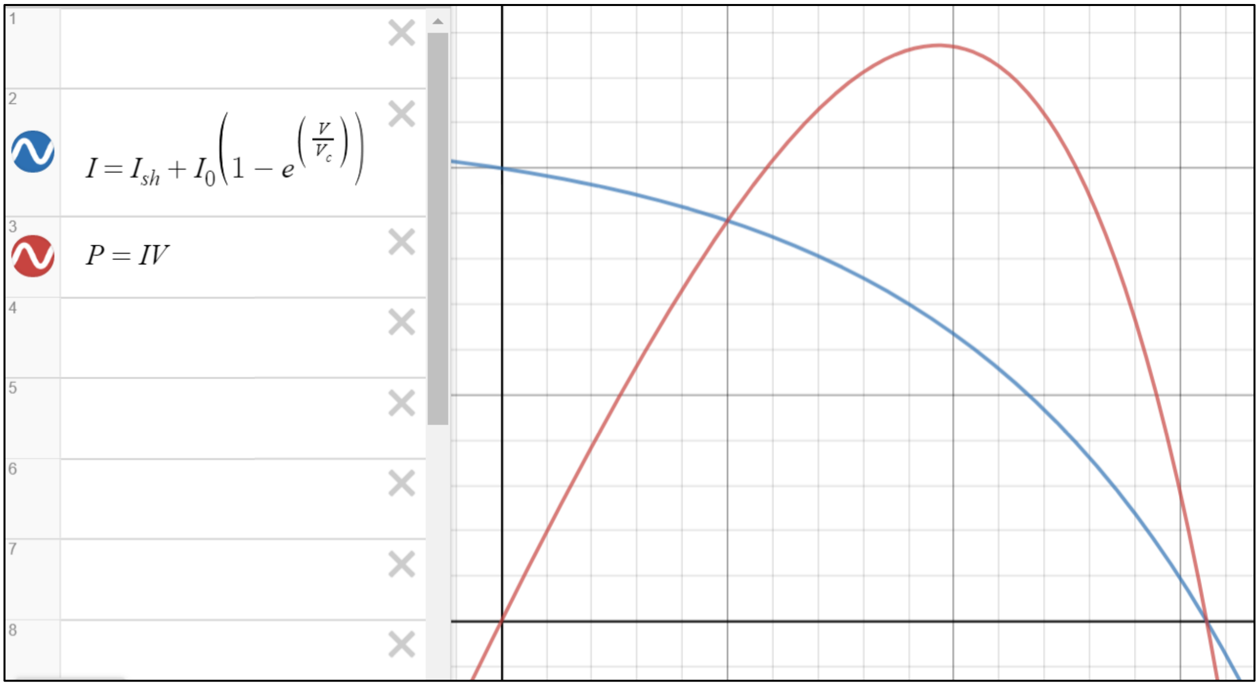
$$I_0 \equiv q\left[R_{g\gamma C} + R_\phi(0)\right] \tag{23}$$

is the reverse saturation current and

$$I_{sc} \equiv q\left[R_{g\gamma L} - R_{g\gamma C}\right] \cong q\left[R_{g\gamma L}\right] \text{ because } R_{g\gamma L} \gg R_{g\gamma C} \tag{24}$$

is the short circuit current of the cell.

Figure 7: Graph of the current and power in PV cell (Eq. 22)



This blue graph shows $I(V)$ as described by Eq. (22). The red graph shows the power output. The intersection of the blue graph with the vertical axis is the short circuit current while the intersection with the horizontal axis is the open circuit voltage.

To properly calculate the practical efficiency of the cell, the maximum output power is needed. This translates to maximizing the *IV* curve shown above (*Figure 7*). Although it is possible to directly calculate the term $I(V_{max})V_{max}$ based on the voltage resulting in maximum power, $V_{\max}$, this will give an answer in terms of some of the not well defined processes like I_0 and eventually $R_\phi(0)$. It is more useful to first calculate $I_{sc}V_{oc}$ in terms of $R_{g\gamma L}$ and then multiply that by the ratio of $\frac{I(V_{\max})V_{\max}}{I_{sc}V_{oc}}$, in which the I_0 will cancel out.

The open circuit voltage, V_{oc}, of the graph of $I(V)$ can be found by setting I equal to zero in Eq. (22).

$$I_{sc} + I_0\left(1 - e^{V_{oc}/V_c}\right) = 0 \tag{25}$$

By solving for V, the open circuit voltage is found.

$$V_{oc} = V_c \ln\left[\frac{I_{sc}}{I_0} + 1\right] \tag{26}$$

Using the definitions for I_0 and I_{sc} in Eq. (23) and (24), the open circuit voltage can be rewritten as

$$V_{oc} = V_c \ln\left[f_\gamma \frac{R_{g\gamma L}}{R_{g\gamma C}} - f_\gamma + 1\right] \tag{27}$$

By using the assumption that $f_\gamma \frac{R_{g\gamma L}}{R_{g\gamma C}} \gg 1 - f_\gamma$ due to the relatively large $R_{g\gamma L}$ and using definitions from Eq. (13) and (14), this can be simplified to the following expression.

$$V_{oc}\left(T, \nu_g, P_L, \nu_0, \delta\nu_0, f\right) \cong V_c \ln\left[f_\gamma \frac{R_{g\gamma L}}{R_{g\gamma C}}\right] = V_c \ln\left[f \frac{Q(\nu_g, S_L)}{Q(\nu_g, S_B)}\right] \tag{28}$$

where

$$f \equiv \frac{f_\gamma f_L}{f_B} \tag{29}$$

Multiplying three fractions, the term f will be a fraction. Since its components are quite hard to compute, it is possible to simply plug in different values for the fraction f instead.

To calculate the nominal efficiency, which is the efficiency if the cell's output is at a maximum possible current and voltage, I_{sc} and V_{oc}, respectively, the input power needs to be written in terms of the ultimate efficiency function. This can be achieved by rearranging Eq. (8) and (9).

$$P_{in} = AP_L = \frac{Ah\nu_g Q\left(\nu_g, S_L\right)}{\eta_{ult}} \tag{30}$$

The nominal efficiency function can then be defined as the following expression.

$$\eta_{nom} = \frac{I_{sc}V_{oc}}{P_{in}} = \frac{qR_{g\gamma L}V_{oc}}{h\nu_g AQ\left(\nu_g, S_L\right)}\eta_{ult} \tag{31}$$

By using the definitions from Eq. (14), this expression can be simplified down to

$$\eta_{nom} = f_L \frac{V_{oc}}{V_g} \eta_{ult} \tag{32}$$

where

$$V_g \equiv \frac{E_g}{q} = \frac{h\nu_g}{q} \tag{33}$$

represents the voltage of the band gap.

The term $\frac{I(V_{\max})V_{\max}}{I_{sc}V_{oc}}$ can now be calculated by maximizing the power graph in *Figure 7*. Since the $P = IV$ curve looks like a classical curve with a maximum value as shown, the voltage that results in the practical maximum power can be found by setting the derivative of the power with respect to the voltage equal to zero.

$$\frac{d}{dV} IV = 0$$

By plugging in $I_{sc} = I_0\left(e^{V_{oc}/V_c} - 1\right)$, derived from Eq. (26), it is possible to calculate $I(V)$ only in terms of I_0, V_{oc}, V_c, and V.

$$I(V) = I_0\left[e^{V_{oc}/V_c} - e^{V/V_c}\right] \tag{34}$$

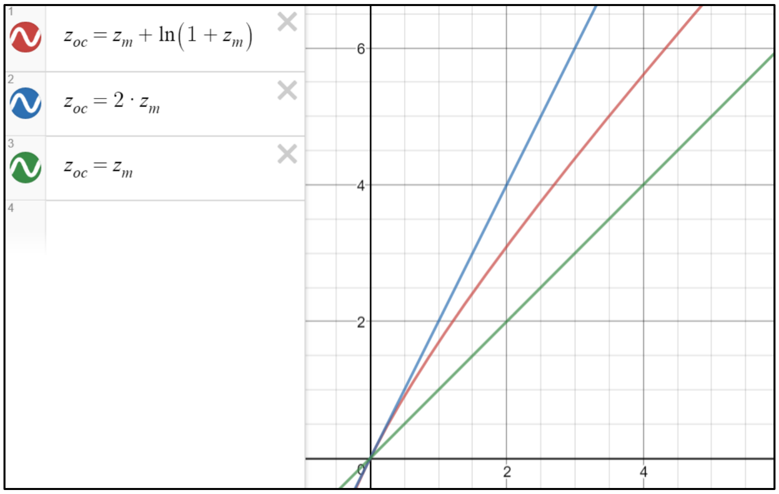
Using this definition, the derivative of the power will be the following expression.

$$\frac{d}{dV} I(V)V = I_0 \left[e^{V_{oc}/V_c} - \left(1 + \frac{V}{V_c}\right) e^{V/V_c}\right] \tag{35}$$

When this derivative is set equal to zero and solved for V, the voltage V becomes $V_{\max}$. By substituting $z_{oc} \equiv \frac{V_{oc}}{V_c}$ and $z_m \equiv \frac{V_{max}}{V_c}$ for ease, as Shockley and Queisser did, a solution to this system is the following expression.

$$z_{oc} = z_m + \ln(1 + z_m) \tag{36}$$

Figure 8: Graph of the relation between z_{oc} and z_m (Eq. 36)



The blue graph shows the relationship that results in V_{max}. The red and green graphs show approximations to the blue graph near zero and extremely high values, respectively.

However, Eq. (36) cannot be solved for z_m, which is what is needed to calculate V_{max}. The following approximations can be utilized in extreme cases.

$$
\begin{aligned}
z_m &\cong \frac{1}{2} z_{oc} \text{ when } V_{oc} \text{ and } V_{max} \ll V_c \\
z_m &\cong z_{oc} \text{ when } V_{oc} \text{ and } V_{max} \gg V_c
\end{aligned}
\tag{37}
$$

However, since these require very drastic conditions, a numerical approximation to solve z_m in Eq. (36) is more practical.

The ratio of maximum power to the power from the short circuit current multiplied by the open circuit voltage, as discussed earlier, can be condensed down to the following equation:

$$m = \frac{I(V_{\max})V_{\max}}{I_{sc}V_{oc}} \tag{38}$$

By plugging in values of the all the terms in this section, the ratio m can be written as a function of z_m.

$$m(z_m) = \frac{{z_m}^2}{(1 + z_m - e^{-z_m})(z_m + \ln(1 + z_m))} \tag{39}$$

If the approximations from Eq. (37) can be made, m can also be written as a closed form function of z_{oc}.

$$m(z_{oc}) \cong \begin{cases} \dfrac{{z_{oc}}^2}{\left(2 + z_{oc} - 2e^{-\frac{z_{oc}}{2}}\right)\left(z_{oc} + 2\ln\left(1 + \frac{z_{oc}}{2}\right)\right)}, & V_{oc} \ll V_c \\ \dfrac{{z_{oc}}^2}{(1 + z_{oc} - e^{-z_{oc}})(z_{oc} + \ln(1 + z_{oc}))}, & V_{oc} \gg V_c \end{cases} \quad (40)$$

To finally arrive at the practical efficiency, derived from finding the current-voltage characteristics of the cell, the nominal efficiency can be multiplied by the ratio m.

$$\eta_{prac} = m\eta_{nom} = \frac{V_{oc}}{V_g} m f_L \eta_{ult} \quad (41)$$

$$\begin{aligned} &\eta_{prac}\left(T, \nu_g, P_L, \nu_0, \delta\nu_0, f_L, f\right) \\ &= f_L \frac{V_{oc}\left(T, \nu_g, P_L, \nu_0, \delta\nu_0, f\right)}{V_g} m\left(T, \nu_g, P_L, \nu_0, \delta\nu_0, f\right)\eta_{ult}\left(\nu_0, \nu_g, \delta\nu_0\right) \end{aligned} \quad (42)$$

The practical efficiency is also a 3 dimensional function of ν_g and ν_0 if the other variables are collapsed to constant values. This 3 dimensional function can be represented with the following color map, similar to the color map for the ultimate efficiency in *Figure 6*.

Figure 9: Color map representation of the practical efficiency function (Eq. 42)

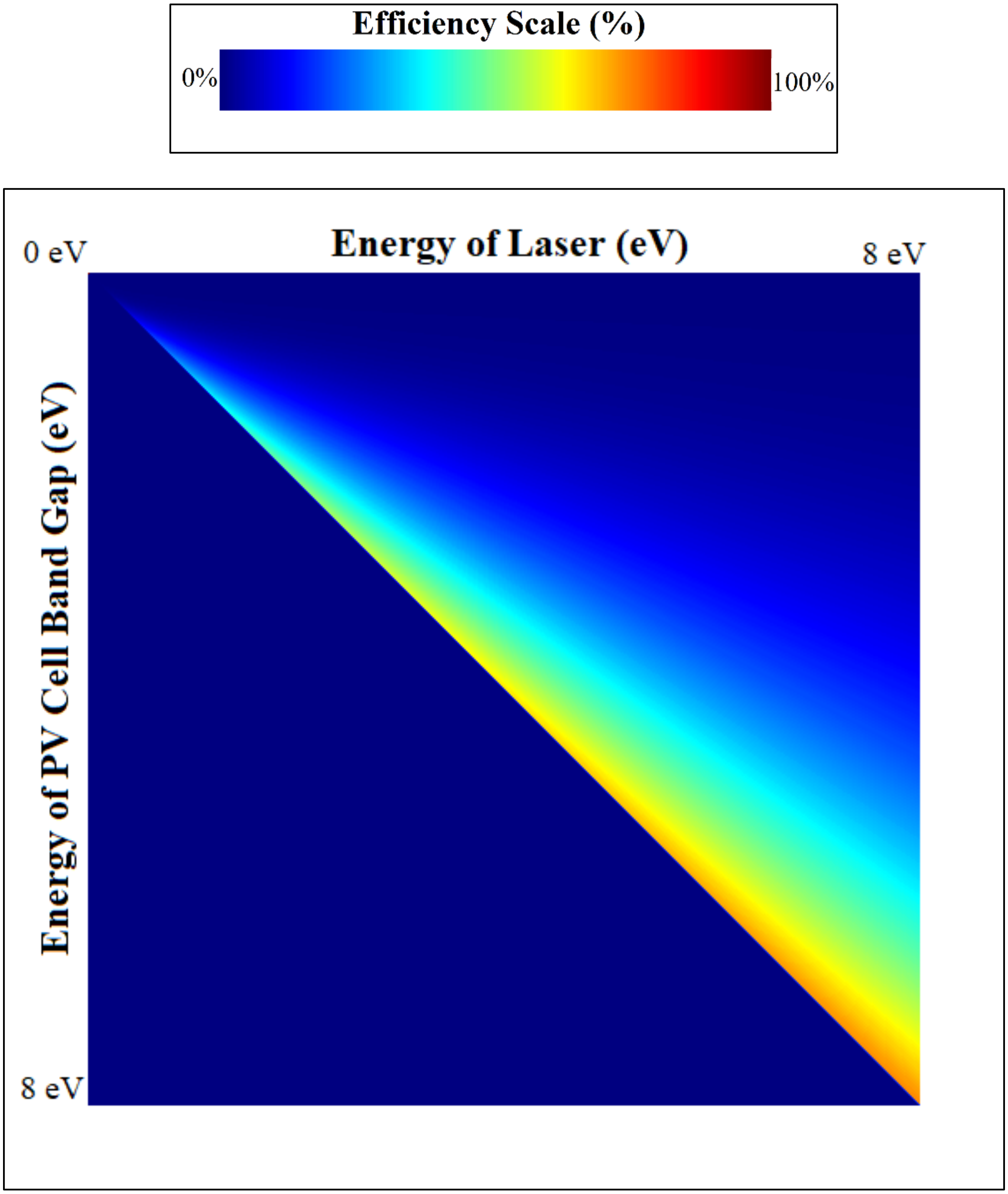


Figure 9 shows a color map representation of the practical efficiency function (Eq. 42) using the scale shown above. The data was found using $T = 300\ K$, $P_L = 100\ \frac{W}{m^2}$, $\delta\nu_0 = 10\ GHz$, $f = f_L = 0.95$.

Results

The following table summarizes the data in the table in the appendix and shows some notably high ultimate and practical efficiencies that result from real laser and PV cell combinations.

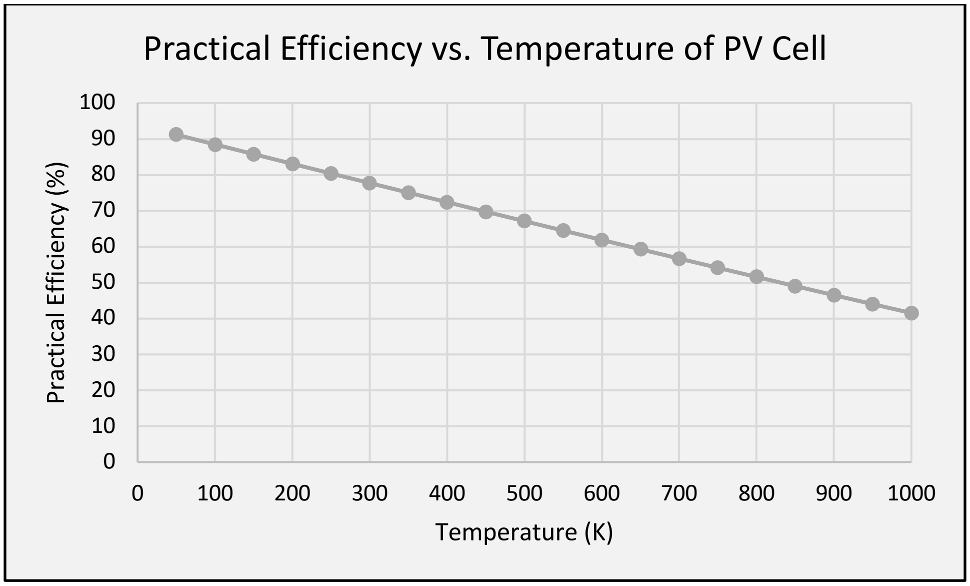
Table 1: Notably high laser-PV cell combination efficiencies

Laser	PV Cell	Ultimate Efficiency	Practical Efficiency
I (0.94 eV)	Cu_2SnS_3 (0.91 eV)	96.51%	56.61%
InGaAsP (1.24 eV)	BAs (1.14 eV)	91.95%	58.68%
Ruby (1.79 eV)	CdSe (1.74 eV)	97.44%	70.27%
Xe+ (2.48 eV)	ZnTe (2.25 eV)	90.74%	74.54%
Xe+ (2.48 eV)	CdS (2.42 eV)	97.59%	80.17%
Ar+ (2.73 eV)	ZnSe (2.7 eV)	98.99%	77.79%
XeF-Excimer (3.51 eV)	TiO_2 (3.2 eV)	91.11%	73.41%
XeF-Excimer (3.51 eV)	ZnO (3.37 eV)	95.95%	77.89%
XeF-Excimer (3.51 eV)	CuCl (3.4 eV)	96.80%	78.68%
XeF-Excimer (3.51 eV)	GaN (3.44 eV)	97.94%	79.73%
N (3.68 eV)	ZnS-cubic (3.54 eV)	96.25%	78.64%
N (3.68 eV)	NiO (3.6 eV)	97.88%	80.15%
XeCl-Excimer (4.03 eV)	ZnS-hex (3.91 eV)	97.13%	80.34%
ArF-Excimer (6.42 eV)	AlN (6.28 eV)	97.76%	84.65%

Table 1 shows the combinations of the listed lasers with the listed direct band gap semiconductors and their resulting ultimate and practical efficiencies. The data was found using $T = 300\ K$, $P_L = 100\ \frac{W}{m^2}$, $\delta\nu_0 = 10\ GHz$, $f = f_L = 0.95$. For a full table of all laser and PV cell ultimate and practical efficiencies calculated, see *Table 2* and *Table 3* in the Appendix. The indirect band gap semiconductors were not considered in the table above due to the previous discussion about how unreliable their processes of creating hole-electron pairs may be.

To see how the practical efficiency changes by altering different variables that affect it, the following graphs were made to see its relationship with temperature, power of the laser, and the fraction f.

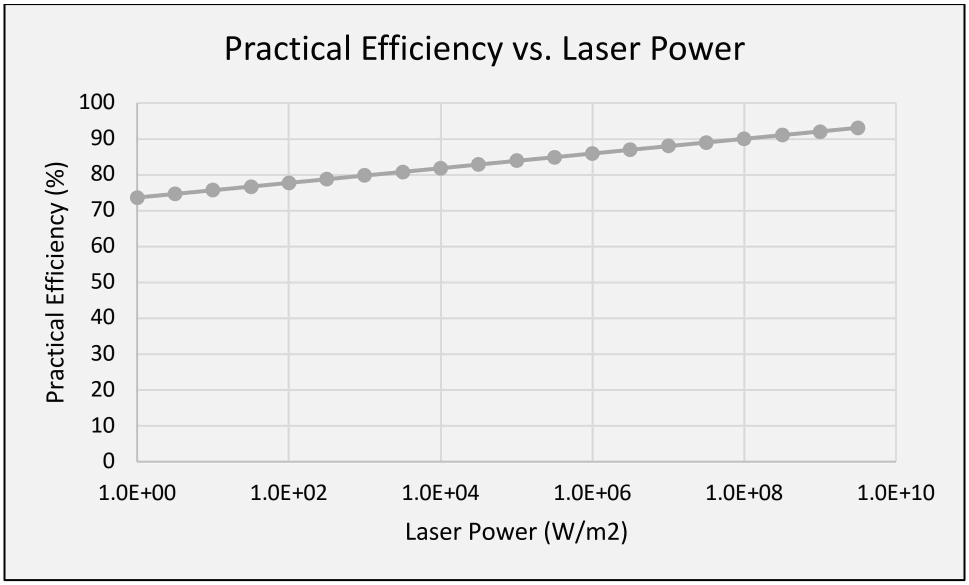
Figure 10: Graph of the practical efficiency versus the temperature of the cell



The data was found using an argon ion laser with a zinc selenide semiconductor and $h\nu_g = 2.7\ eV$, $P_L = 100\ \frac{W}{m^2}$, $h\nu_0 = 2.73\ eV$, $\delta\nu_0 = 10\ GHz$, $f = f_L = 0.95$.

The practical efficiency decreases linearly as the temperature increases. This is because the higher the thermal activity within the cell, the higher the rates of non-radiative recombination and generation and radiative recombination. More of the energy coming from the laser is wasted as heat and other non-useable sources, decreasing the open circuit voltage. The efficiency may seem to reach below 0% at a high enough temperature, but before that temperature, non-linear effects will take over, invalidating the model described in this paper.

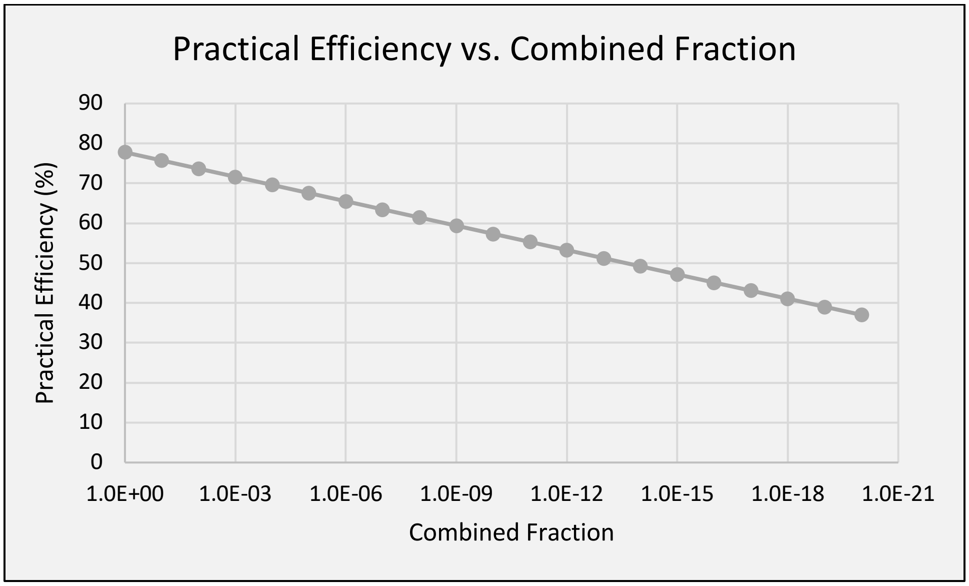
Figure 11: Graph of the practical efficiency versus the power of the laser



The data was found using an argon ion laser with a zinc selenide semiconductor and $T = 300\ K$, $h\nu_g = 2.7\ eV$, $h\nu_0 = 2.73\ eV$, $\delta\nu_0 = 10\ GHz$, $f = f_L = 0.95$.

The practical efficiency increases linearly as the laser power increases exponentially. This is because of the definition of V_{oc} in Eq. (28). As the power of the laser increases, the non-radiative recombination and generation rates due to the temperature of the cell as well as the radiative recombination rate becomes more insignificant compared to the radiative generation caused by the laser. Although it seems that with enough laser power, the practical efficiency will exceed 100%, at a certain point before that, similar to this model under high temperatures, non-linear effects will take place, invalidating the model described in this paper.

Figure 12: Graph of the practical efficiency versus the fraction, f



The data was found using an argon ion laser with a zinc selenide semiconductor and $T = 300\ K$, $h\nu_g = 2.7\ eV$, $P_L = 100\ \frac{W}{m^2}$, $h\nu_0 = 2.73\ eV$, $\delta\nu_0 = 10\ GHz$, $f_L = 0.95$.

The practical efficiency decreases linearly as the fraction f decreases exponentially. This is also because of the definition of V_{oc} in Eq. (28). As the fraction decreases to less orders of magnitude, there is a smaller percentage of the total rates of generation and recombination that are radiative. This has an effect of how much of the energy is wasted through non-useful sources, decreasing the efficiency.

Conclusion

It is theoretically possible to achieve efficiencies above 90% when transmitting energy with lasers and practical efficiencies of above 80% should also be obtainable. One of the highest resulting efficiencies comes from the combination of an argon fluoride excimer laser with a PV cell made from aluminum nitride semiconductor. Some limiting factors that decrease the efficiency may be lack of proper laser light-cell incidence, electrical losses within the cell and the circuit it is connected to, and other non-linear quantum effects. Future research should look into these factors through more intensive theoretical calculations and construct a more accurate model as well as do real life tests on the efficiencies of the notable combinations of lasers and PV cells to see how high real efficiencies can reach. Overall, the minimization of the limiting factors coupled with the right combination of laser and semiconductor can potentially make laser power beaming and power over fiber optics into viable techniques of energy transfer in the future.

References Cited

1. Hoffman JM. To infinity ... and beyond! Mach Des 2007 Mar 08.

2. Beam it up; The Economist 2011 Mar 12;398(8724):5-SS6.

3. Vision-guided laser beaming powers remote vehicles. Vision Systems Design 2011 10;16(10):15.

4. Ditmire T. High-power lasers. Am Sci 2010 Sep;98(5):394-401.

5. Hecht J. Photonic power conversion delivers power via laser beams. Laser Focus World 2006 01;42(1):113.

6. Raible DE. Free space optical communications with high intensity laser power beaming. Ann Arbor: Cleveland State University; 2011. 221 p:Copyright - Database copyright ProQuest LLC; ProQuest does not claim copyright in the individual underlying works; Last updated - 2016-03-10.

7. Piesing M. A bright new idea for flight. The Independent. 2011 Apr 26;Sect. 10.

8. Treusch G, Koenning T, Shelef B. High-power diode lasers boost power-beaming competition. Laser Focus World 2008 03;44(3):75,77,79.

9. William Shockley and Hans Queisser. 1961. Detailed Balance Limit of Efficiency of p-n Junction Solar Cells. Journal of Applied Physics.

10. Peter W. Milonni and Joseph H Eberly. 1988. Lasers.

Appendix

Table 2: Ultimate Efficiencies of Various Laser-PV Cell Combinations

	CO_2 (0.12 eV)	CO (0.48 eV)	Iodine (0.94 eV)	InGaAsP (1.24 eV)	Ruby (1.79 eV)	HeNe (1.96 eV)	Xe+ (2.48 eV)	Ar+ (2.73 eV)	Kr+ (2.98 eV)	XeF-Excimer (3.51 eV)	N (3.68 eV)	XeCl-Excimer (4.03 eV)	KrF-Excimer (5.0 eV)	KrCl-Excimer (5.58 eV)	ArF-Excimer (6.42 eV)	F2-Excimer (7.9 eV)	Sm (169.84 eV)
Direct Band Gap Semiconductors																	
InSb (0.17 eV)	0.0%	35.65%	18.03%	13.71%	9.52%	8.68%	6.86%	6.23%	5.7%	4.84%	4.62%	4.22%	3.4%	3.04%	2.65%	2.15%	0.1%
PbSe (0.27 eV)	0.0%	56.62%	28.64%	21.78%	15.12%	13.78%	10.89%	9.9%	9.06%	7.69%	7.34%	6.71%	5.4%	4.83%	4.2%	3.42%	0.16%
InAs (0.36 eV)	0.0%	75.49%	38.18%	29.04%	20.16%	18.37%	14.52%	13.2%	12.08%	10.25%	9.79%	8.94%	7.2%	6.45%	5.6%	4.56%	0.21%
InN (0.7 eV)	0.0%	0.0%	74.24%	56.46%	39.2%	35.73%	28.23%	25.67%	23.49%	19.93%	19.03%	17.39%	14.0%	12.53%	10.9%	8.86%	0.41%
GaSb (0.73 eV)	0.0%	0.0%	77.0%	58.56%	40.66%	37.05%	29.28%	26.62%	24.36%	20.67%	19.74%	18.04%	14.52%	13.0%	11.3%	9.19%	0.43%
Cu_2SnS_3 (0.91 eV)	0.0%	0.0%	96.51%	73.4%	50.96%	46.45%	36.7%	33.37%	30.53%	25.91%	24.74%	22.61%	18.2%	16.29%	14.17%	11.52%	0.54%
$CuInSe_2$ (1.0 eV)	0.0%	0.0%	0.0%	80.65%	56.0%	51.04%	40.33%	36.67%	33.55%	28.47%	27.19%	24.84%	20.0%	17.91%	15.57%	12.66%	0.59%
BAs (1.14 eV)	0.0%	0.0%	0.0%	91.95%	63.84%	58.18%	45.97%	41.8%	38.25%	32.46%	31.0%	28.32%	22.8%	20.41%	17.75%	14.44%	0.67%
SnS (1.3 eV)	0.0%	0.0%	0.0%	0.0%	72.8%	66.35%	52.43%	47.67%	43.62%	37.01%	35.35%	32.29%	26.0%	23.28%	20.24%	16.46%	0.77%
InP (1.35 eV)	0.0%	0.0%	0.0%	0.0%	75.6%	68.9%	54.44%	49.5%	45.3%	38.44%	36.71%	33.54%	27.0%	24.17%	21.01%	17.09%	0.79%
Zn_3P_2 (1.5 eV)	0.0%	0.0%	0.0%	0.0%	84.0%	76.56%	60.49%	55.0%	50.33%	42.71%	40.78%	37.26%	30.0%	26.86%	23.35%	18.99%	0.88%
GaAs (1.43 eV)	0.0%	0.0%	0.0%	0.0%	80.08%	72.99%	57.67%	52.43%	47.98%	40.71%	38.88%	35.52%	28.6%	25.6%	22.26%	18.11%	0.84%
CdTe (1.49 eV)	0.0%	0.0%	0.0%	0.0%	83.44%	76.05%	60.09%	54.63%	49.99%	42.42%	40.51%	37.01%	29.8%	26.68%	23.19%	18.87%	0.88%
CdSe (1.74 eV)	0.0%	0.0%	0.0%	0.0%	97.44%	88.81%	70.17%	63.8%	58.38%	49.54%	47.31%	43.22%	34.8%	31.16%	27.09%	22.03%	1.02%
ZnTe (2.25 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	90.74%	82.5%	75.49%	64.06%	61.18%	55.89%	45.01%	40.29%	35.02%	28.49%	1.32%
CdS (2.42 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	97.59%	88.73%	81.2%	68.9%	65.8%	60.12%	48.41%	43.33%	37.67%	30.64%	1.42%
ZnSe (2.7 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	98.99%	90.59%	76.87%	73.41%	67.07%	54.01%	48.34%	42.03%	34.19%	1.59%
TiO_2 (3.2 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	91.11%	87.0%	79.49%	64.01%	57.3%	49.81%	40.52%	1.88%
ZnO (3.37 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	95.95%	91.63%	83.72%	67.41%	60.34%	52.46%	42.67%	1.98%
CuCl (3.4 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	96.8%	92.44%	84.46%	68.01%	60.88%	52.93%	43.05%	2.0%
GaN (3.44 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	97.94%	93.53%	85.46%	68.81%	61.59%	53.55%	43.56%	2.03%
ZnS-cubic (3.54 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	96.25%	87.94%	70.81%	63.39%	55.11%	44.83%	2.08%
NiO (3.6 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	97.88%	89.43%	72.01%	64.46%	56.04%	45.59%	2.12%
ZnS-hex (3.91 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	97.13%	78.21%	70.01%	60.86%	49.51%	2.3%
AlN (6.28 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	97.76%	79.52%	3.7%
Indirect Band Gap Semiconductors																	
Sn (0.08 eV)	68.39%	16.78%	8.48%	6.45%	4.48%	4.08%	3.23%	2.93%	2.68%	2.28%	2.18%	1.99%	1.6%	1.43%	1.25%	1.01%	0.05%
Ge (0.67 eV)	0.0%	0.0%	71.06%	54.04%	37.52%	34.2%	27.02%	24.57%	22.48%	19.08%	18.22%	16.64%	13.4%	12.0%	10.43%	8.48%	0.39%
Si (1.12 eV)	0.0%	0.0%	0.0%	90.33%	62.72%	57.16%	45.17%	41.07%	37.58%	31.89%	30.45%	27.82%	22.4%	20.05%	17.43%	14.18%	0.66%
Cu_2S (1.2 eV)	0.0%	0.0%	0.0%	96.78%	67.2%	61.25%	48.39%	44.0%	40.26%	34.17%	32.63%	29.81%	24.0%	21.49%	18.68%	15.2%	0.71%
MoS_2 (1.23 eV)	0.0%	0.0%	0.0%	99.19%	68.88%	62.78%	49.6%	45.1%	41.27%	35.02%	33.44%	30.56%	24.6%	22.02%	19.15%	15.58%	0.72%
BP (2.0 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	80.66%	73.33%	67.11%	56.94%	54.38%	49.68%	40.01%	35.81%	31.13%	25.33%	1.18%
GaSe (2.1 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	84.69%	77.0%	70.46%	59.79%	57.1%	52.17%	42.01%	37.6%	32.69%	26.59%	1.24%
AlAs (2.16 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	87.11%	79.2%	72.47%	61.5%	58.73%	53.66%	43.21%	38.68%	33.62%	27.35%	1.27%
GaP (2.26 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	91.14%	82.86%	75.83%	64.35%	61.45%	56.14%	45.21%	40.47%	35.18%	28.62%	1.33%
SiC (3.3 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	93.95%	89.72%	81.98%	66.01%	59.09%	51.37%	41.79%	1.94%
$B_{12}As_2$ (3.47 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	98.79%	94.34%	86.2%	69.41%	62.13%	54.02%	43.94%	2.04%
C (5.47 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	97.94%	85.15%	69.27%	3.22%
BN-hex (5.96 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	92.78%	75.47%	3.51%
BN-cubic (6.36 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	99.0%	80.54%	3.74%

Table 2 shows all the efficiencies of the combinations of the listed lasers with the listed semiconductors and their resulting ultimate efficiencies. The data was found using $T = 300\ K$, $P_L = 100\ \frac{W}{m^2}$, $\delta\nu_0 = 10\ GHz$, $f = f_L = 0.95$.

Table 3: Practical Efficiencies of Various Laser-PV Cell Combinations

	CO_2 (0.12 eV)	CO (0.48 eV)	Iodine (0.94 eV)	InGaAsP (1.24 eV)	Ruby (1.79 eV)	HeNe (1.96 eV)	Xe+ (2.48 eV)	Ar+ (2.73 eV)	Kr+ (2.98 eV)	XeF-Excimer (3.51 eV)	N (3.68 eV)	XeCl-Excimer (4.03 eV)	KrF-Excimer (5.0 eV)	KrCl-Excimer (5.58 eV)	ArF-Excimer (6.42 eV)	F2-Excimer (7.9 eV)	Sm (169.84 eV)
Direct Band Gap Semiconductors																	
InSb (0.17 eV)	0.0%	1.69%	0.25%	0.04%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
PbSe (0.27 eV)	0.0%	10.62%	4.15%	2.81%	1.64%	1.42%	1.12%	0.85%	0.74%	0.57%	0.53%	0.45%	0.31%	0.26%	0.2%	0.13%	0.0%
InAs (0.36 eV)	0.0%	22.54%	9.94%	7.11%	4.54%	4.04%	3.31%	2.67%	2.39%	1.94%	1.83%	1.63%	1.23%	1.06%	0.89%	0.67%	0.01%
InN (0.7 eV)	0.0%	0.0%	37.79%	28.23%	19.13%	17.33%	14.61%	12.17%	11.07%	9.28%	8.84%	8.02%	6.36%	5.65%	4.86%	3.89%	0.14%
GaSb (0.73 eV)	0.0%	0.0%	40.09%	29.97%	20.34%	18.43%	15.55%	12.96%	11.79%	9.9%	9.42%	8.56%	6.79%	6.03%	5.19%	4.17%	0.15%
Cu2SnS3 (0.91 eV)	0.0%	0.0%	56.61%	42.55%	29.06%	26.37%	22.32%	18.66%	17.01%	14.32%	13.65%	12.42%	9.9%	8.81%	7.61%	6.13%	0.24%
CuInSe2 (1.0 eV)	0.0%	0.0%	0.01%	48.82%	33.41%	30.34%	25.7%	21.51%	19.61%	16.53%	15.76%	14.34%	11.45%	10.2%	8.82%	7.11%	0.29%
BAs (1.14 eV)	0.0%	0.0%	0.0%	58.68%	40.26%	36.58%	31.02%	25.99%	23.71%	20.01%	19.08%	17.38%	13.89%	12.39%	10.72%	8.66%	0.36%
SnS (1.3 eV)	0.0%	0.0%	0.0%	0.01%	48.18%	43.8%	37.17%	31.17%	28.46%	24.04%	22.92%	20.89%	16.72%	14.92%	12.92%	10.44%	0.44%
InP (1.35 eV)	0.0%	0.0%	0.0%	0.01%	50.67%	46.07%	39.11%	32.8%	29.95%	25.3%	24.13%	21.99%	17.61%	15.71%	13.61%	11.01%	0.47%
Zn3P2 (1.5 eV)	0.0%	0.0%	0.0%	0.0%	58.18%	52.91%	44.94%	37.72%	34.45%	29.12%	27.78%	25.32%	20.29%	18.11%	15.69%	12.7%	0.55%
GaAs (1.43 eV)	0.0%	0.0%	0.0%	0.0%	54.67%	49.71%	42.22%	35.42%	32.34%	27.33%	26.07%	23.77%	19.03%	16.99%	14.72%	11.91%	0.51%
CdTe (1.49 eV)	0.0%	0.0%	0.0%	0.0%	57.67%	52.45%	44.55%	37.39%	34.14%	28.86%	27.53%	25.1%	20.11%	17.95%	15.55%	12.59%	0.54%
CdSe (1.74 eV)	0.0%	0.0%	0.0%	0.0%	70.27%	63.94%	54.35%	45.65%	41.7%	35.27%	33.66%	30.7%	24.61%	21.98%	19.06%	15.44%	0.67%
ZnTe (2.25 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	74.54%	62.68%	57.28%	48.49%	46.28%	42.23%	33.9%	30.3%	26.29%	21.32%	0.95%
CdS (2.42 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	80.17%	68.39%	62.51%	52.93%	50.52%	46.1%	37.02%	33.09%	28.71%	23.29%	1.04%
ZnSe (2.7 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	77.79%	71.15%	60.26%	57.52%	52.5%	42.17%	37.7%	32.72%	26.55%	1.19%
TiO2 (3.2 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	73.41%	70.08%	63.97%	51.41%	45.97%	39.91%	32.4%	1.46%
ZnO (3.37 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	77.89%	74.36%	67.89%	54.56%	48.79%	42.36%	34.4%	1.56%
CuCl (3.4 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	78.68%	75.11%	68.58%	55.11%	49.29%	42.8%	34.75%	1.57%
GaN (3.44 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	79.73%	76.12%	69.5%	55.86%	49.95%	43.37%	35.22%	1.59%
ZnS-cubic (3.54 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.04%	78.64%	71.8%	57.71%	51.61%	44.82%	36.39%	1.65%
NiO (3.6 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.01%	80.15%	73.19%	58.83%	52.61%	45.69%	37.1%	1.68%
ZnS-hex (3.91 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	80.34%	64.59%	57.77%	50.17%	40.75%	1.85%
AlN (6.28 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	84.65%	68.81%	3.16%
Indirect Band Gap Semiconductors																	
Sn (0.08 eV)	0.71%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Ge (0.67 eV)	0.0%	0.0%	35.17%	26.24%	17.75%	16.07%	13.54%	11.26%	10.24%	8.58%	8.17%	7.41%	5.87%	5.21%	4.48%	3.58%	0.13%
Si (1.12 eV)	0.0%	0.0%	0.0%	57.27%	39.28%	35.69%	30.26%	25.35%	23.13%	19.51%	18.6%	16.94%	13.54%	12.07%	10.45%	8.43%	0.35%
Cu2S (1.2 eV)	0.0%	0.0%	0.0%	62.93%	43.22%	39.28%	33.32%	27.93%	25.49%	21.52%	20.52%	18.69%	14.95%	13.33%	11.54%	9.33%	0.39%
MoS2 (1.23 eV)	0.0%	0.0%	0.0%	65.0%	44.7%	40.63%	34.48%	28.9%	26.38%	22.27%	21.24%	19.35%	15.48%	13.81%	11.95%	9.66%	0.41%
BP (2.0 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.02%	64.62%	54.3%	49.62%	41.99%	40.07%	36.56%	29.33%	26.21%	22.73%	18.43%	0.81%
GaSe (2.1 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.01%	68.59%	57.65%	52.68%	44.59%	42.55%	38.82%	31.16%	27.84%	24.15%	19.58%	0.87%
AlAs (2.16 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	70.97%	59.66%	54.52%	46.15%	44.04%	40.18%	32.25%	28.82%	25.01%	20.28%	0.9%
GaP (2.26 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	74.93%	63.01%	57.59%	48.76%	46.53%	42.46%	34.08%	30.46%	26.43%	21.44%	0.95%
SiC (3.3 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	76.05%	72.59%	66.27%	53.26%	47.63%	41.35%	33.58%	1.52%
B12As2 (3.47 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	80.52%	76.88%	70.19%	56.41%	50.45%	43.81%	35.57%	1.61%
C (5.47 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	83.84%	72.84%	59.19%	2.71%
BN-hex (5.96 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	79.99%	65.01%	2.98%
BN-cubic (6.36 eV)	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	85.81%	69.76%	3.2%

Table 3 shows all the efficiencies of the combinations of the listed lasers with the listed semiconductors and their resulting efficiencies. The data was found using $T = 300\ K$, $P_L = 100\ \frac{W}{m^2}$, $\delta\nu_0 = 10\ GHz$, $f = f_L = 0.95$.

The following is all of the code that was used in this project. It contains all the numerical calculations computed to make *Table 2* and *Table 3* as well as the color graphs in *Figure 6* and *Figure 9* and the graphs of *Figure 10*, *Figure 11* and *Figure 12*.

```
import java.awt.Color;
import java.awt.image.BufferedImage;
import java.io.File;
import java.io.FileOutputStream;
import java.io.IOException;
import java.io.PrintStream;
import java.math.BigDecimal;
import java.math.RoundingMode;
import java.util.HashMap;
import java.util.LinkedHashMap;
import java.util.Scanner;

import javax.imageio.ImageIO;

public class Main {

      // public static final double M_TO_NM = Math.pow(10, -9);
      public static final double J_TO_eV = 6.241509343260179 * Math.pow(10,
18);

      public static final double H_IN_J_S = 6.62607004081 * Math.pow(10, -
34);
      public static final double H_IN_eV_S = H_IN_J_S * J_TO_eV;

      public static final double C_IN_M_PER_S = 299792458;

      public static final double K_IN_J_PER_K = 1.3806485279 * Math.pow(10, -
23);

      public static final double Q_IN_C = 1.602176620898 * Math.pow(10, -19);

      public static void main(String[] args) throws Exception {
            calculate();
```

```
    }

    public static void calculate() {

        double T = 300;
        double vg = 2.7 / H_IN_eV_S;

        double P_L = 1E2;
        double v0 = 2.73 / H_IN_eV_S;
        double a = 1E10;
        double f_L = .95;
        double f = .95;

        double Q_L = Q_L(P_L, v0, vg, a);
        double Q_B = ApproximateBlackBodyIntegral.Q_B(T, vg);

        double V_c = V_c(T);
        double V_op = V_op(V_c, f, Q_L, Q_B);
        double V_g = V_g(vg);

        double z_op = z_op(V_op, V_c);

        double z_m = ApproximateZm.z_m(z_op);
        double m = m(z_m);

        double V_m = V_m(z_m, V_c);

        double ult = ultimateEfficiency(vg, v0, a);
        double nom = nominalEfficiency(T, vg, P_L, v0, a, f_L, f);
        double prac = practicalEfficiency(T, vg, P_L, v0, a, f_L, f);

        System.out.println("Q_L: " + Q_L);
        System.out.println("Q_B: " + Q_B);

        System.out.println("V_c: " + V_c);
        System.out.println("V_g: " + V_g);
        System.out.println("V_op: " + V_op);
```

```
            System.out.println("z_op: " + z_op);
            System.out.println("z_m: " + z_m);
            System.out.println("V_m: " + V_m);

            System.out.println("m: " + m);

            System.out.println("ult: " + ult);
            System.out.println("nom: " + nom);
            System.out.println("prac: " + prac);

            tableOfUltimateEfficiencies(T, vg, P_L, v0, a, f_L, f);
            tableOfPracticalEfficiencies(T, vg, P_L, v0, a, f_L, f);
            colorMapUltimateEfficiencies(T, vg, P_L, v0, a, f_L, f);
            colorMapPracticalEfficiencies(T, vg, P_L, v0, a, f_L, f);

            graphTemperature(vg, P_L, v0, a, f_L, f);
            graphLaserPower(T, vg, v0, a, f_L, f);
            graphFraction(T, vg, P_L, v0, a, f_L);
      }

      public static void graphTemperature(double vg, double P_L, double v0,
double a, double f_L, double f) {
            try {
                  File outputFile = new File("effVsT.txt");
                  outputFile.createNewFile();
                  PrintStream outFileStream = new PrintStream(new
FileOutputStream(outputFile, false), true);

                  String str = "";

                  for (int T = 50; T <= 1000; T += 50) {
                        double eff = practicalEfficiency(T, vg, P_L, v0, a,
f_L, f);
                        // System.out.println("T: " + T + " eff: " + eff);
                        str += T + "\t" + eff + "\r\n";
                  }
```

```
                    outFileStream.print(str);
                    outFileStream.close();
              } catch (Exception e) {

              }
       }

       public static void graphLaserPower(double T, double vg, double v0,
double a, double f_L, double f) {
              try {
                    File outputFile = new File("effVsP_L.txt");
                    outputFile.createNewFile();
                    PrintStream outFileStream = new PrintStream(new
FileOutputStream(outputFile, false), true);

                    String str = "";

                    for (double order = 0; order < 10; order += .5) {
                          double P_L = Math.pow(10, order);
                          double eff = practicalEfficiency(T, vg, P_L, v0, a,
f_L, f);
                          // System.out.println("P_L: " + P_L + " eff: " +
eff);
                          str += P_L + "\t" + eff + "\r\n";
                    }

                    outFileStream.print(str);
                    outFileStream.close();
              } catch (Exception e) {

              }
       }

       public static void graphFraction(double T, double vg, double P_L,
double v0, double a, double f_L) {
              try {
                    File outputFile = new File("effVsf.txt");
                    outputFile.createNewFile();
```

```
                  PrintStream outFileStream = new PrintStream(new
FileOutputStream(outputFile, false), true);

                  String str = "";

                  for (double f = 1E-20; f <= 2; f *= 10) {
                        double eff = practicalEfficiency(T, vg, P_L, v0, a,
f_L, f);
                        // System.out.println("f: " + f + " eff: " + eff);
                        str += f + "\t" + eff + "\r\n";
                  }

                  outFileStream.print(str);
                  outFileStream.close();
            } catch (Exception e) {

            }
      }

      public static void tableOfUltimateEfficiencies(double T, double vg,
double P_L, double v0, double a, double f_L,
                  double f) {

      LaserSemiconductorValues.calculateRealValues("ultimateEfficiencies",
new TwoDPercentFunction() {

                  @Override
                  public double calculate(double v0, double vg) {
                        return ultimateEfficiency(vg, v0, a);
                  }
            });
      }

      public static void tableOfPracticalEfficiencies(double T, double vg,
double P_L, double v0, double a, double f_L,
                  double f) {

      LaserSemiconductorValues.calculateRealValues("practicalEfficiencies",
new TwoDPercentFunction() {
```

```
                  @Override
                  public double calculate(double v0, double vg) {
                        return practicalEfficiency(T, vg, P_L, v0, a, f_L,
f);
                  }
            });
      }

      public static void colorMapUltimateEfficiencies(double T, double vg,
double P_L, double v0, double a, double f_L,
                  double f) {

            ColorImageWriter.writeImage(500, 500, "ultEffGrayHeatMap", new
TwoDPercentFunction() {

                  @Override
                  public double calculate(double v0, double vg) {
                        double ans = ultimateEfficiency(vg, v0, a);
                        return ans;
                  }
            });

      }

      public static void colorMapPracticalEfficiencies(double T, double vg,
double P_L, double v0, double a, double f_L,
                  double f) {

            ColorImageWriter.writeImage(500, 500, "practEffGrayHeatMap", new
TwoDPercentFunction() {

                  @Override
                  public double calculate(double v0, double vg) {
                        double ans = practicalEfficiency(T, vg, P_L, v0, a,
f_L, f);
                        return ans;
                  }
            });
```

```
	}

	public static double ultimateEfficiency(double vg, double v0, double a)
{
		double answer = 1;
		answer *= vg;
		answer /= (Math.PI + 2 * Math.atan(v0 / a));
		answer /= (Math.pow(a, 2) + Math.pow(v0, 2));

		double mainfunc = 0;

		mainfunc += (Math.PI * v0);
		mainfunc += (a * Math.log(1 - 2 * v0 / vg + (Math.pow(a, 2) +
Math.pow(v0, 2)) / Math.pow(vg, 2)));
		mainfunc += (2 * v0 * Math.atan((v0 - vg) / a));

		answer *= mainfunc;

		return answer;
	}

	public static double nominalEfficiency(double T, double vg, double P_L,
double v0, double a, double f_L, double f) {

		double Q_L = Q_L(P_L, v0, vg, a);
		double Q_B = ApproximateBlackBodyIntegral.Q_B(T, vg);

		double V_c = V_c(T);
		double V_op = V_op(V_c, f, Q_L, Q_B);
		double V_g = V_g(vg);

		double ans = voltageFraction(V_op, V_g) * f_L *
ultimateEfficiency(vg, v0, a);
		if (ans < 1)
			return ans;
		else
			return 0;
	}
```

```
      public static double practicalEfficiency(double T, double vg, double
P_L, double v0, double a, double f_L,
                  double f) {
            try {
                  if (vg == 0 || v0 == 0)
                        return 0;
                  double Q_L = Q_L(P_L, v0, vg, a);
                  double Q_B = ApproximateBlackBodyIntegral.Q_B(T, vg);

                  double V_c = V_c(T);
                  double V_op = V_op(V_c, f, Q_L, Q_B);
                  double V_g = V_g(vg);

                  double z_op = z_op(V_op, V_c);

                  double z_m = ApproximateZm.z_m(z_op);

                  return m(z_m) * nominalEfficiency(T, vg, P_L, v0, a, f_L,
f);
            } catch (Exception e) {
                  return 0;
            }
      }

      public static double Q_L(double P_L, double v0, double vg, double a) {
            double ans = 0;
            ans = P_L / (2 * Math.PI * H_IN_J_S);
            ans /= (Math.pow(v0, 2) + Math.pow(a, 2));
            double mainfunc = 0;

            mainfunc += Math.PI * v0;
            mainfunc += a * Math.log((Math.pow(vg - v0, 2) + Math.pow(v0, 2))
/ Math.pow(vg, 2));
            mainfunc += +2 * v0 * Math.atan((v0 - vg) / a);

            ans *= mainfunc;
            return ans;
      }
```

```
	public static double V_op(double V_c, double f, double Q_L, double Q_B)
{
		return V_c * Math.log(f * Q_L / Q_B);
	}

	public static double V_c(double T) {
		double V_c = K_IN_J_PER_K * T / Q_IN_C;
		return V_c;
	}

	public static double V_g(double vg) {
		double V_g = H_IN_J_S * vg / Q_IN_C;
		return V_g;
	}

	public static double z_op(double V_op, double V_c) {
		double z_op = V_op / V_c;
		return z_op;
	}

	public static double V_m(double z_m, double V_c) {
		return z_m * V_c;
	}

	public static double m(double z_m) {
		double ans = Math.pow(z_m, 2);
		ans /= (1 + z_m - Math.exp(-z_m));
		ans /= (z_m + Math.log(1 + z_m));
		return ans;
	}

	public static double voltageFraction(double V_op, double V_g) {
		return V_op / V_g;
	}

	public static String toString(double d) {
		d = round(d, 2);
```

```
		return d + "";
	}

	public static double round(double value, int places) {
		if (places < 0)
			throw new IllegalArgumentException();

		BigDecimal bd = new BigDecimal(value);
		bd = bd.setScale(places, RoundingMode.HALF_UP);
		return bd.doubleValue();
	}

}

class ApproximateZm {
	// solves x+ln(1+x)-z_op=0
	// x+ln(1+x)=z_op
	public static double z_m(Double z_op) {
		if (z_op < 0 || z_op.isInfinite() || z_op.isNaN())
			throw new RuntimeException("invalid: " + z_op);
		return solve(z_op, 0, z_op);
	}

	private static double solve(double z_op, double a, double b) {
		double c = (a + b) / 2;
		double c_ans = c + Math.log(c + 1);

		if (percentError(c_ans, z_op) < .0005)
			return c;
		else if (c_ans > z_op)
			return solve(z_op, a, c);
		else
			return solve(z_op, c, b);
	}

	private static double percentError(double est, double val) {
		return Math.abs(est - val) / val;
```

```
	}
}

class ApproximateBlackBodyIntegral extends Main {

	public static double Q_B(double T, double vg) {
		double wavelength = C_IN_M_PER_S / vg;
		double ans = integral(T, 0, wavelength, 1000);
		return ans;
	}

	private static double integral(double T, double startWavelength, double
endWavelength, double numRect) {
		double rectWidth = endWavelength / numRect;

		double ans = 0;
		for (int i = 0; i < numRect; i++) {
			double xpos1 = i * rectWidth + startWavelength;
			double xpos2 = (i + 1) * rectWidth + startWavelength;
			double midxpos = (xpos1 + xpos2) / 2;

			double val = blackBodyAt(midxpos, T);

			ans += (val * rectWidth);
		}
		return ans;
	}

	private static double blackBodyAt(double wavelength, double T) {
		double exp = Math.exp((H_IN_J_S * C_IN_M_PER_S) / (K_IN_J_PER_K *
T * wavelength));
		double ans = 1 / (exp - 1);
		ans /= Math.pow(wavelength, 4);
		ans *= 2 * Math.PI * C_IN_M_PER_S;
		return ans;
	}
}
```

```
class ColorImageWriter extends Main {
      public static void writeImage(int w, int h, String name, 
TwoDPercentFunction function) {
            String path = name + ".png";
            BufferedImage image = new BufferedImage(w, h, 
BufferedImage.TYPE_BYTE_GRAY);
            double maxeff = 0;
            int maxcol = 0;
            for (int y = 0; y < h; y++) {
                  for (int x = 0; x < w; x++) {
                        double laserv = ((double) x / w * 8) / H_IN_eV_S;
                        double bgv = ((double) y / h * 8) / H_IN_eV_S;
                        double efficiency = function.calculate(laserv, bgv);
                        if (efficiency > maxeff) {
                              maxeff = efficiency;
                        }
                        int col = (int) (efficiency * 255);
                        if (col > maxcol) {
                              maxcol = col;
                        }
                        Color color = new Color(col, col, col);
                        image.setRGB(x, y, color.getRGB());
                  }
                  System.out.println((double) y / h);
            }
            System.out.println(maxeff);
            System.out.println(maxcol);

            File ImageFile = new File(path);
            try {
                  ImageIO.write(image, "png", ImageFile);
            } catch (IOException e) {
                  e.printStackTrace();
            }
      }

      public static void imggg() {
            try {
```

```
                  int width = 500, height = 500;
                  int[] pixels1D = new int[width * height];

                  for (int y = 0; y < height; y++) {
                        for (int x = 0; x < width; x++) {
                              double v0eV = (double) x / width * 8;
                              double vgeV = (double) y / height * 8;

                              double v0 = v0eV / Main.H_IN_eV_S;
                              double vg = vgeV / Main.H_IN_eV_S;

                              double efficiency = Main.ultimateEfficiency(vg,
v0, Math.pow(10, 9));

                              int pixel = (int) (efficiency * 255);

                              pixels1D[500 * y + x] = new Color(pixel, pixel,
pixel).getRGB();

                        }
                  }

                  BufferedImage pixelImage = new BufferedImage(width, height,
BufferedImage.TYPE_INT_RGB);
                  pixelImage.setRGB(0, 0, width, height, pixels1D, 0, width);

                  File outFile = new File("test.png");
                  outFile.createNewFile();
                  ImageIO.write(pixelImage, "png", outFile);

            } catch (Exception e) {
            }
      }

}

abstract class TwoDPercentFunction {
```

```
      public abstract double calculate(double a, double b);
}


class LaserSemiconductorValues extends Main {
      public static final File semiconductorFile = new File(

      "C:\\Users\\akars\\Documents\\FIRM\\FIRMProgram\\semiconductors.txt");
      public static final File lasersFile = new
File("C:\\Users\\akars\\Documents\\FIRM\\FIRMProgram\\lasers.txt");
      public static final HashMap<String, Double> lasers = new
LinkedHashMap<String, Double>();
      public static final HashMap<String, Double> semiconductors = new
LinkedHashMap<String, Double>();
      static {


            try {
                  Scanner scanSemiconductors = new
Scanner(semiconductorFile);
                  Scanner scanLasers = new Scanner(lasersFile);


                  while (scanLasers.hasNextLine()) {
                        String[] strs = scanLasers.nextLine().split(" ");
                        try {
                              lasers.put(strs[0],
Double.parseDouble(strs[1]));
                        } catch (Exception e) {
                        }
                  }


                  while (scanSemiconductors.hasNextLine()) {
                        String[] strs = scanSemiconductors.nextLine().split("
");
                        try {
                              semiconductors.put(strs[0],
Double.parseDouble(strs[1]));
                        } catch (Exception e) {
                        }
                  }


                  scanSemiconductors.close();
```

```
                scanLasers.close();
            } catch (Exception e) {
            }


      }


      public static void calculateRealValues(String fileName,
TwoDPercentFunction function) {
            try {
                  File outputFile = new File(fileName + ".txt");

                  String tabstr = "\t";

                  for (String laser : lasers.keySet()) {
                        double eV = H_IN_eV_S * C_IN_M_PER_S /
lasers.get(laser);


                        String eVStr = toString(eV);
                        String laserstr = laser + " (" + eVStr + " eV)";

                        tabstr += laserstr + "\t";
                  }
                  tabstr += "\r\n";

                  double maxeff = 0;
                  String semi = "";
                  String laser_ = "";

                  for (String semiconductor : semiconductors.keySet()) {

                        double bandgap = semiconductors.get(semiconductor);
                        double vg = bandgap / H_IN_eV_S;

                        String bandgapStr = toString(bandgap);
                        tabstr += semiconductor + " (" + bandgapStr + " eV)"
+ "\t";


                        for (String laser : lasers.keySet()) {
```

```
                              double wavelength = lasers.get(laser);
                              double v0 = C_IN_M_PER_S / wavelength;

                              double efficiency = function.calculate(v0, vg);

                              if (efficiency > maxeff) {
                                    maxeff = efficiency;
                                    semi = semiconductor;
                                    laser_ = laser;
                              }
                              String effstr = toString(efficiency * 100);

                              tabstr += effstr + "%\t";
                        }
                        tabstr += "\r\n";
                  }
                  System.out.println(maxeff + " " + laser_ + " " + semi);

                  outputFile.createNewFile();
                  PrintStream outFileStream = new PrintStream(new
FileOutputStream(outputFile, false), true);

                  outFileStream.print(tabstr);
                  // System.out.println(tabstr);

                  outFileStream.close();
            } catch (Exception e) {
            }
      }
}
```